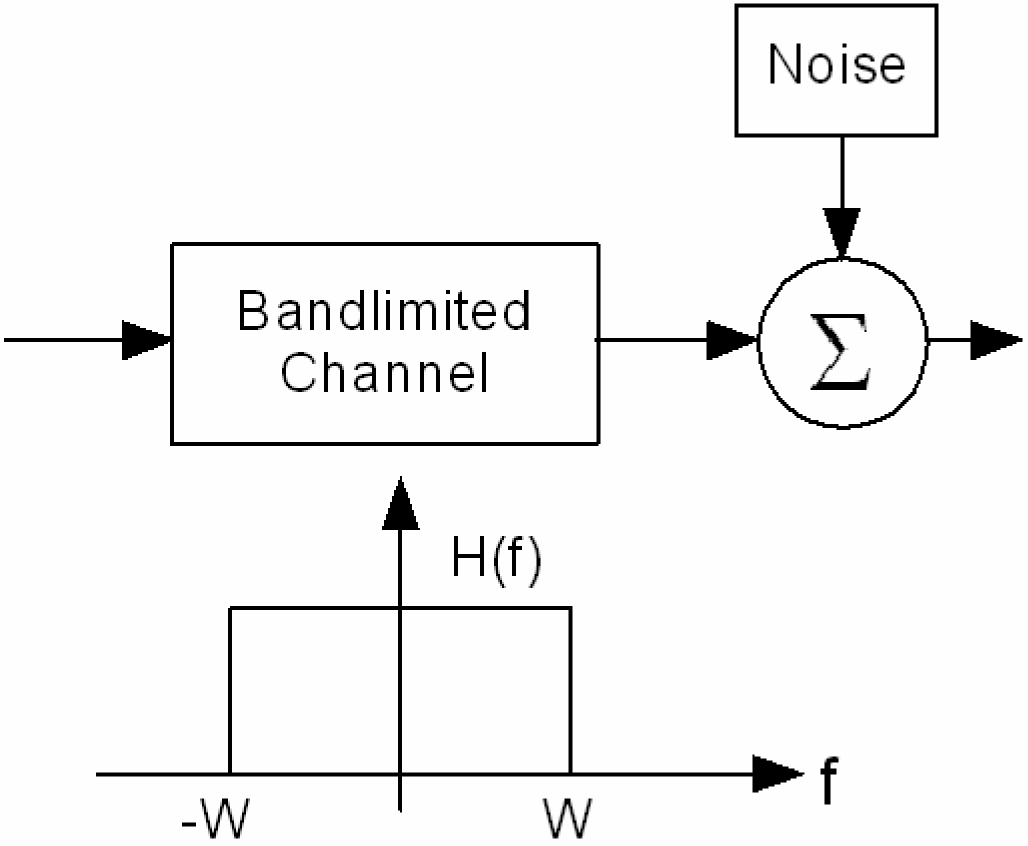
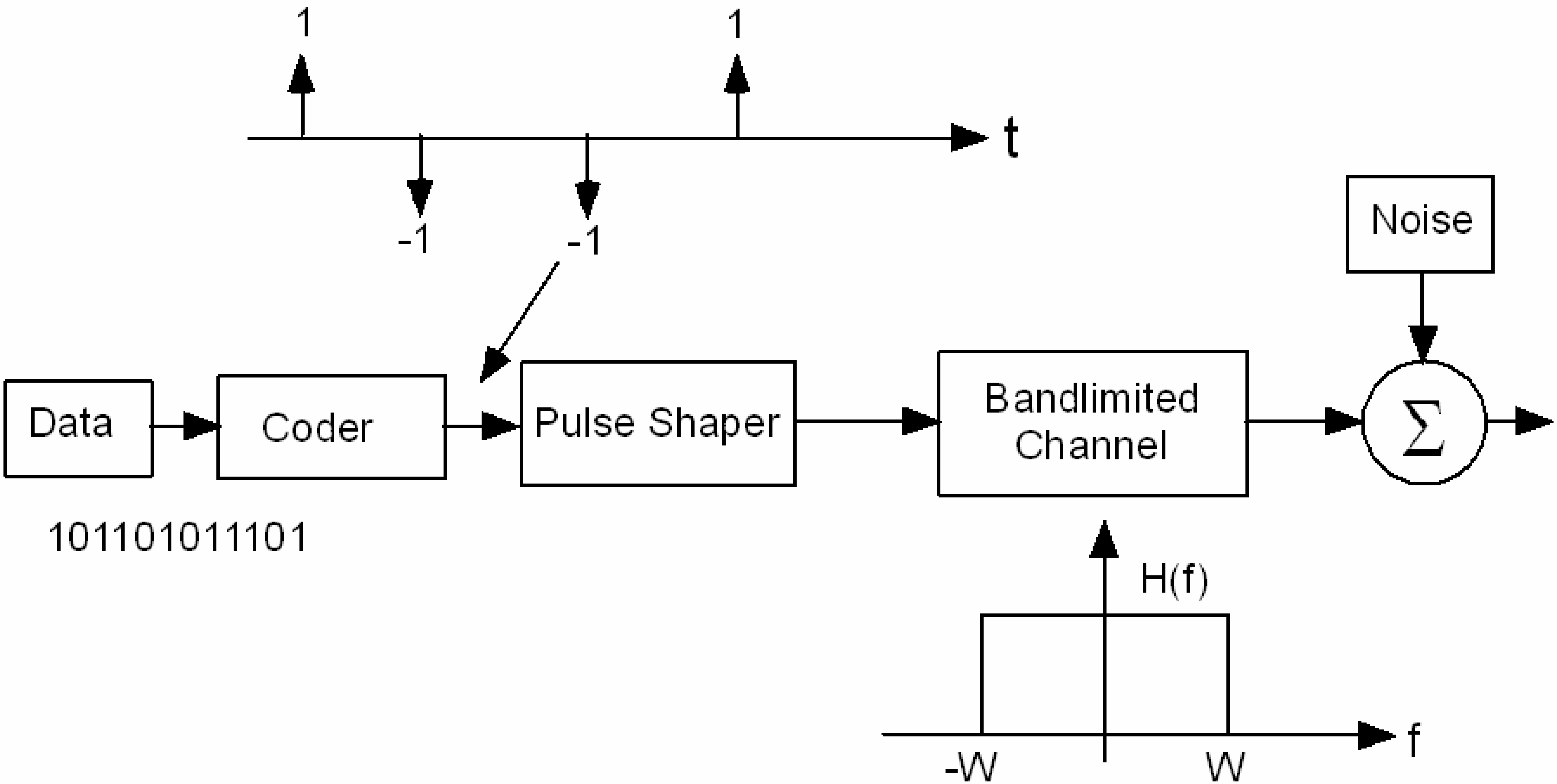
Digital Communications Basics

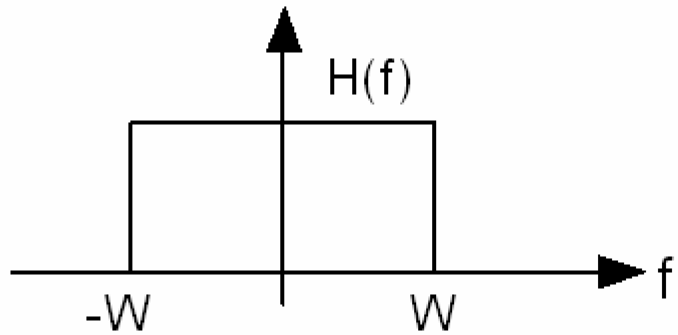
Silicon DSP Corporation

- Power
- Bandwidth
- Fading Multip
- Noise
- Jammers



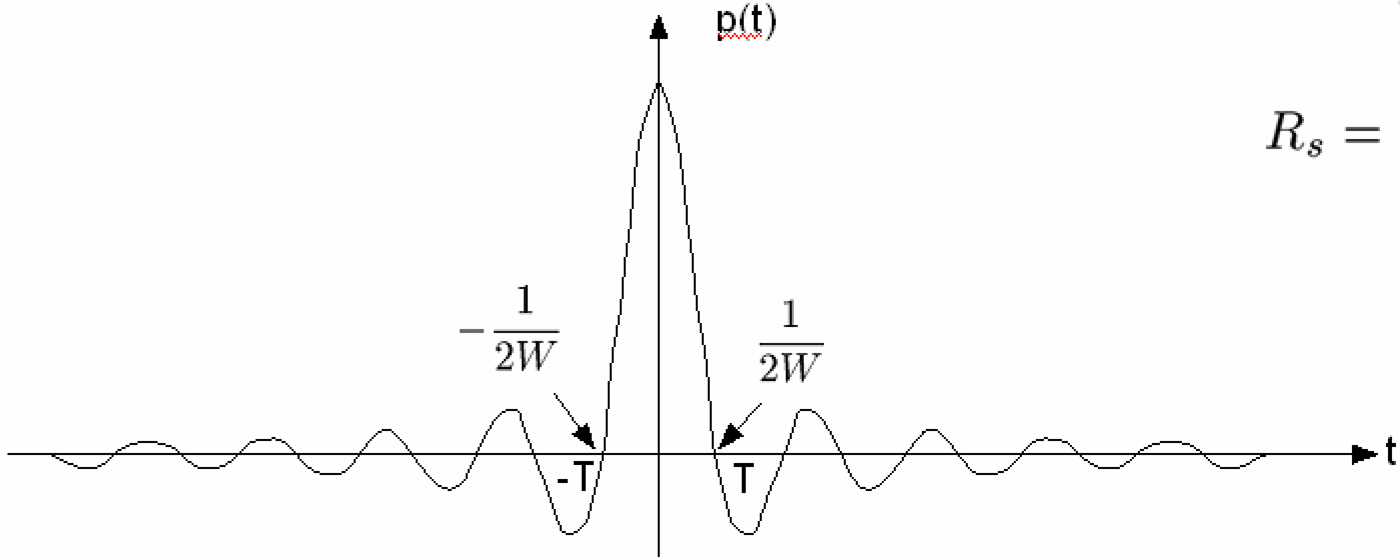

1
1
t
-1
-1
Noise
Data
Coder
Pulse Shaper
Bandlimited
Channel
Σ
101101011101
H(f)
f
-W
W

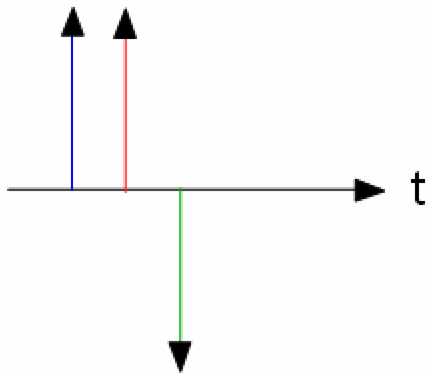
Pulse Shaping for Bandlimited Channel

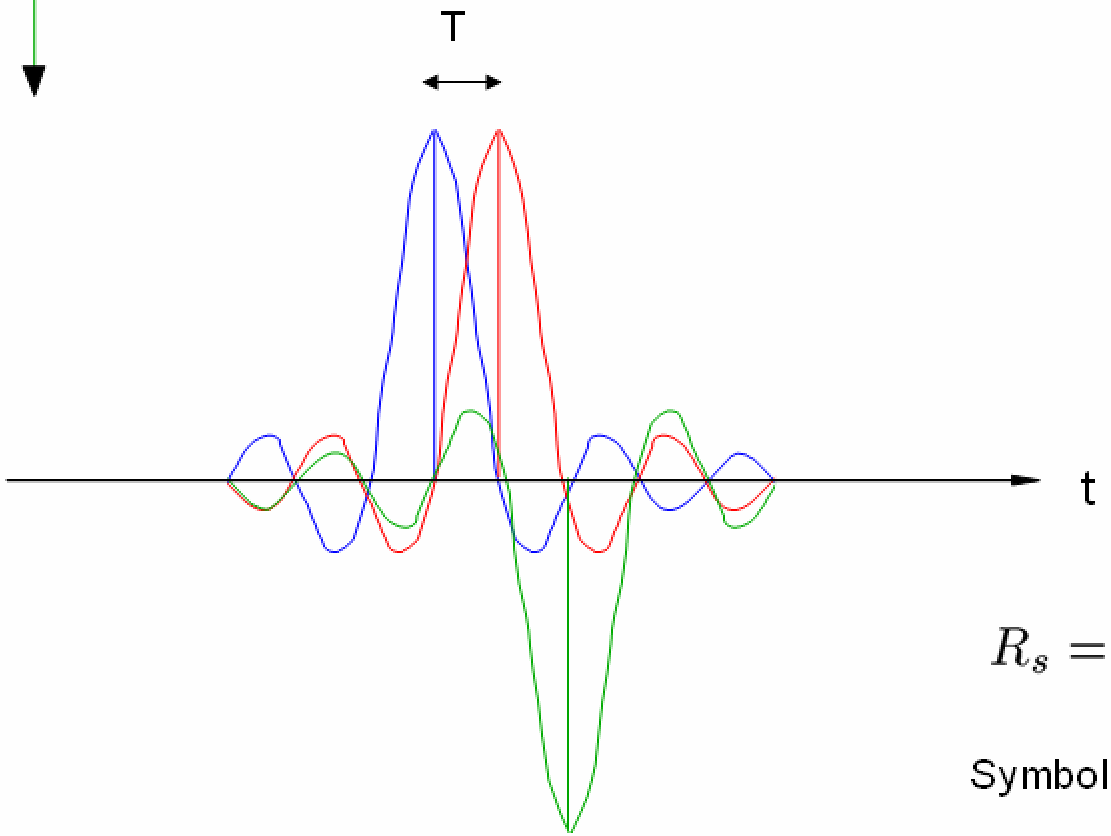


$$T = \frac{1}{2W}$$

$$R_s = \frac{1}{T}$$





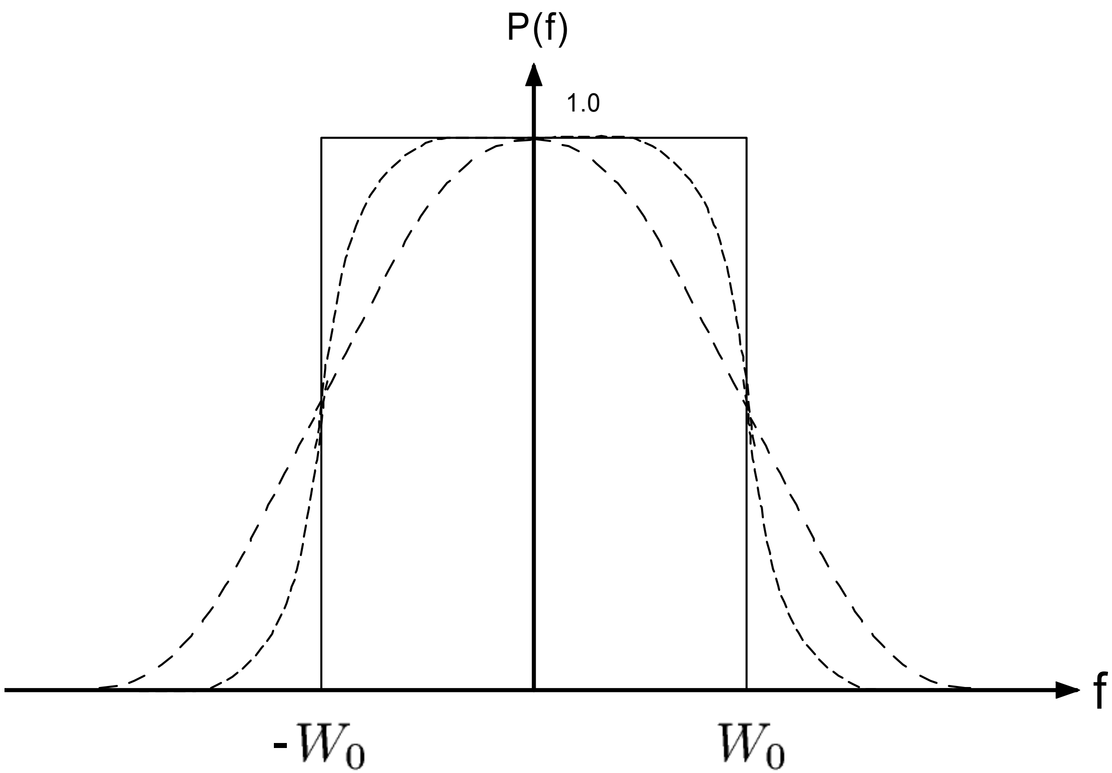


$$R_s = \frac{1}{T}$$

Symbol Rate

Raised Cosine Nyquist Pulse Shaping

$$H(f) = \begin{cases} 1 & \text{for } |f| < 2W_0 - W \\ cos^2\left(\frac{\pi}{4}\frac{|f| + W - 2W_0}{W - W_0}\right) & \text{for } 2W_0 - W < |f| < W \\ 0 & \text{for } |f| < W \end{cases}$$

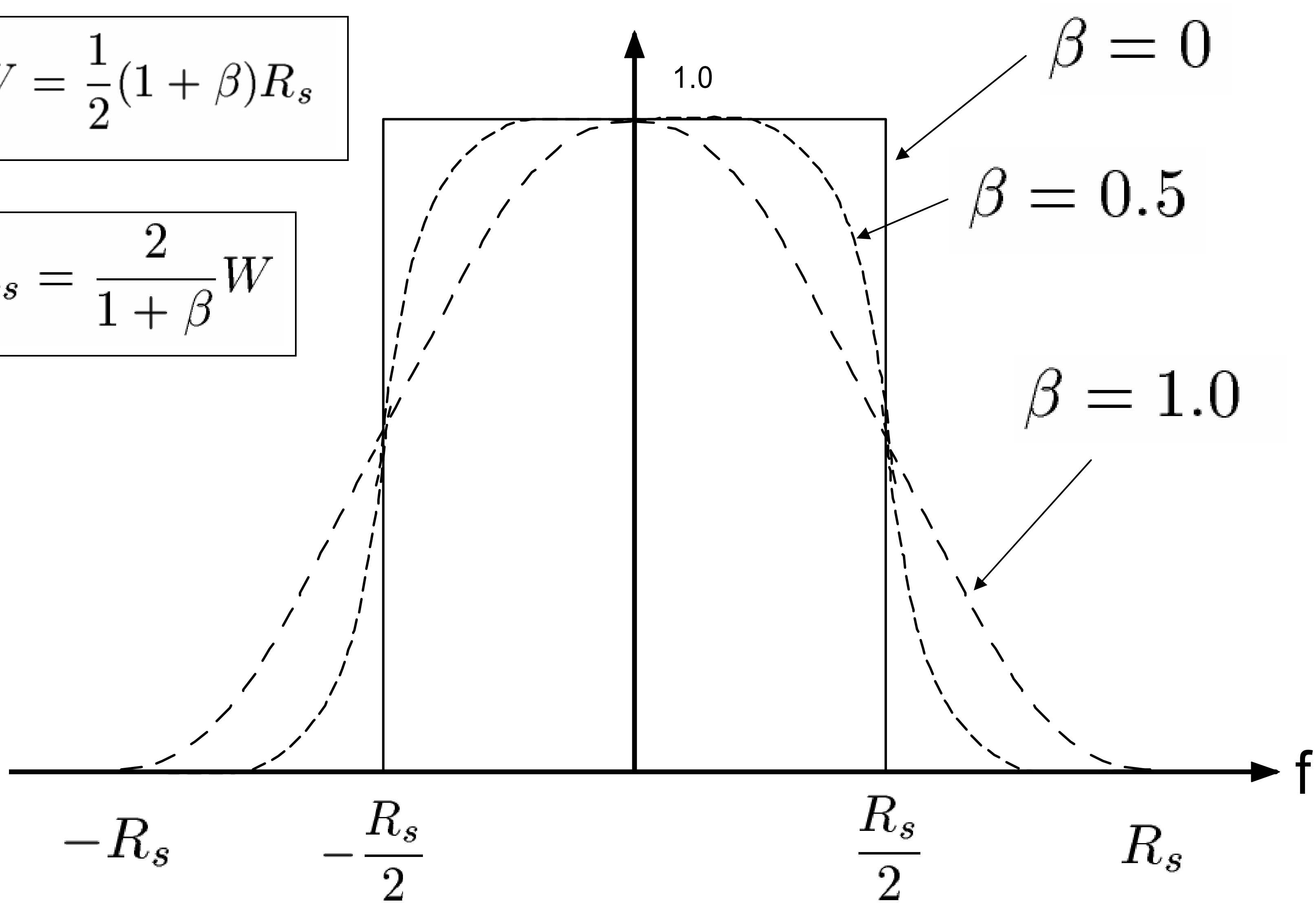


Source:

Bernard Sklar, Digital Communications, Prentice Hall, 2001

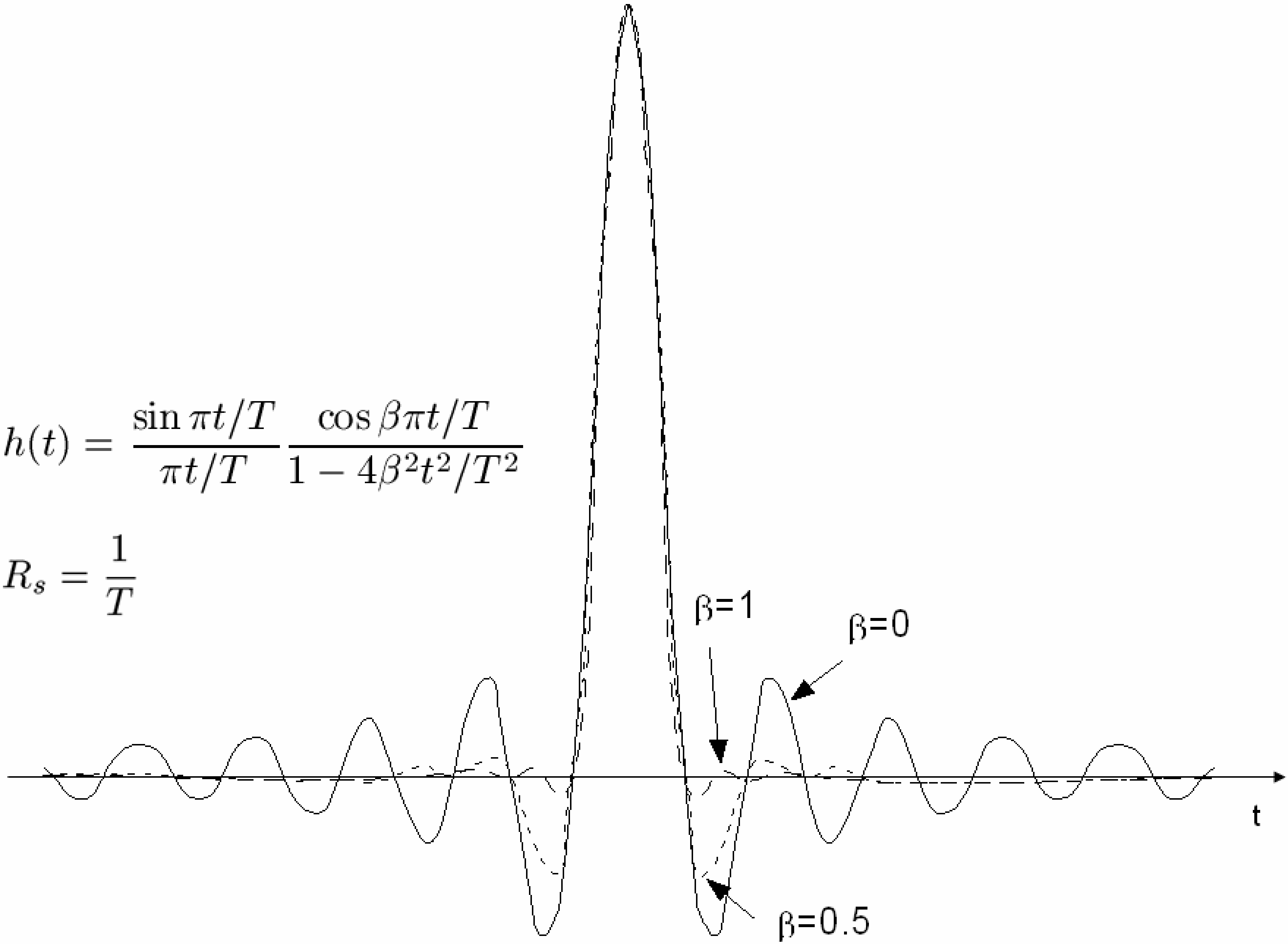
$$W = \frac{1}{2}(1+\beta)R_s$$

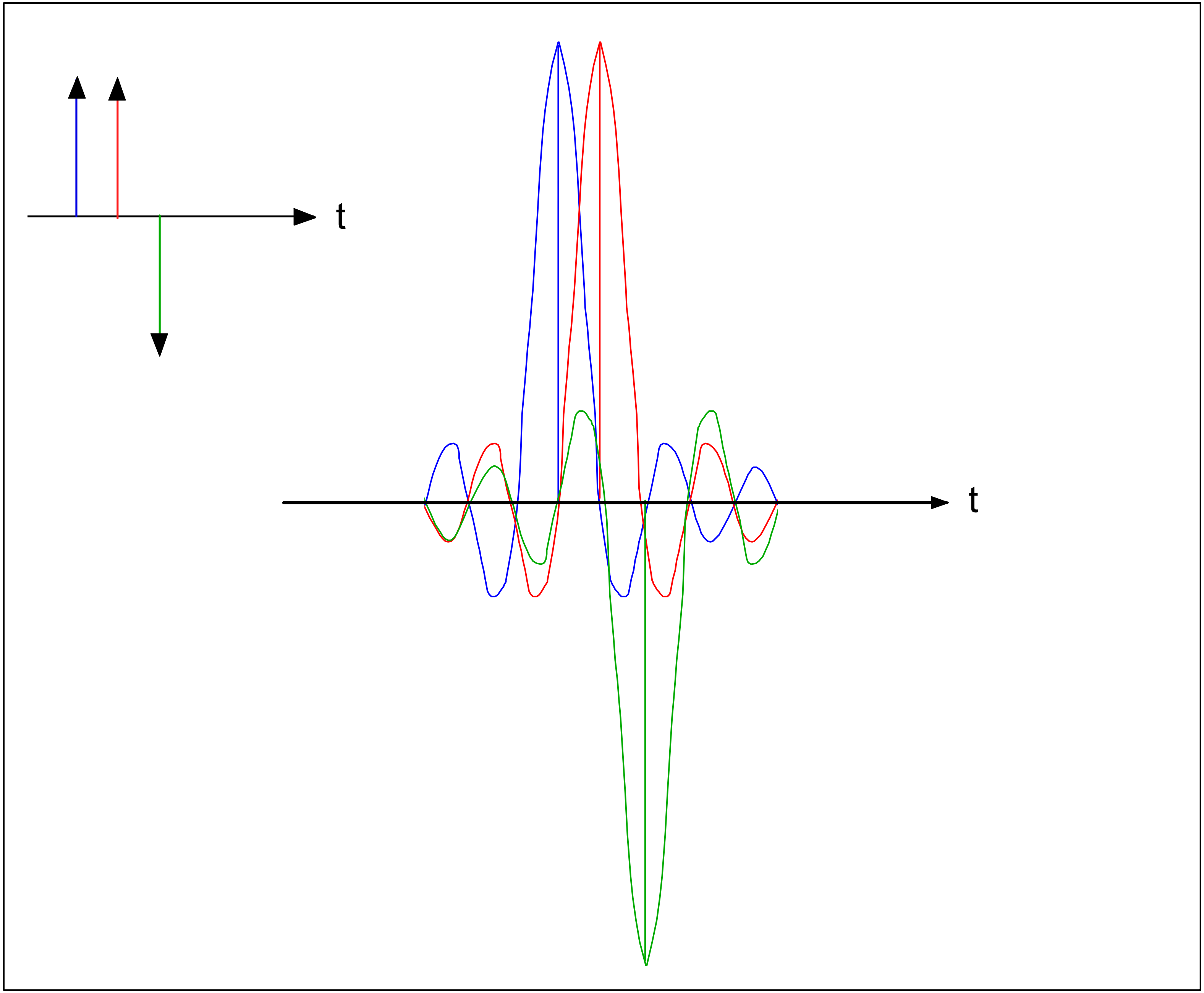
$$R_s = \frac{2}{1+\beta}W$$

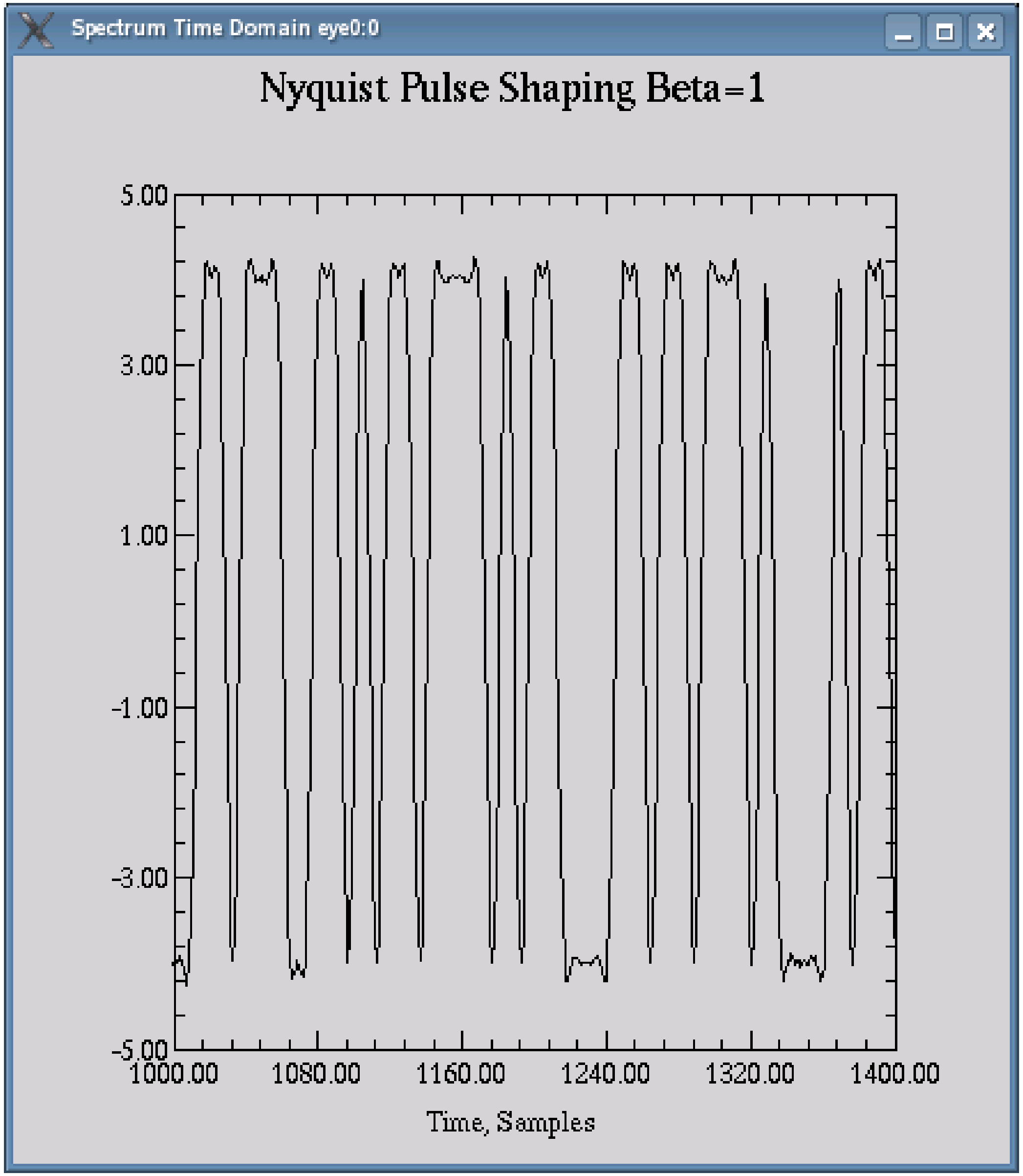


$$h(t) = \frac{\sin \pi t/T}{\pi t/T} \frac{\cos \beta \pi t/T}{1 - 4\beta^2 t^2/T^2}$$

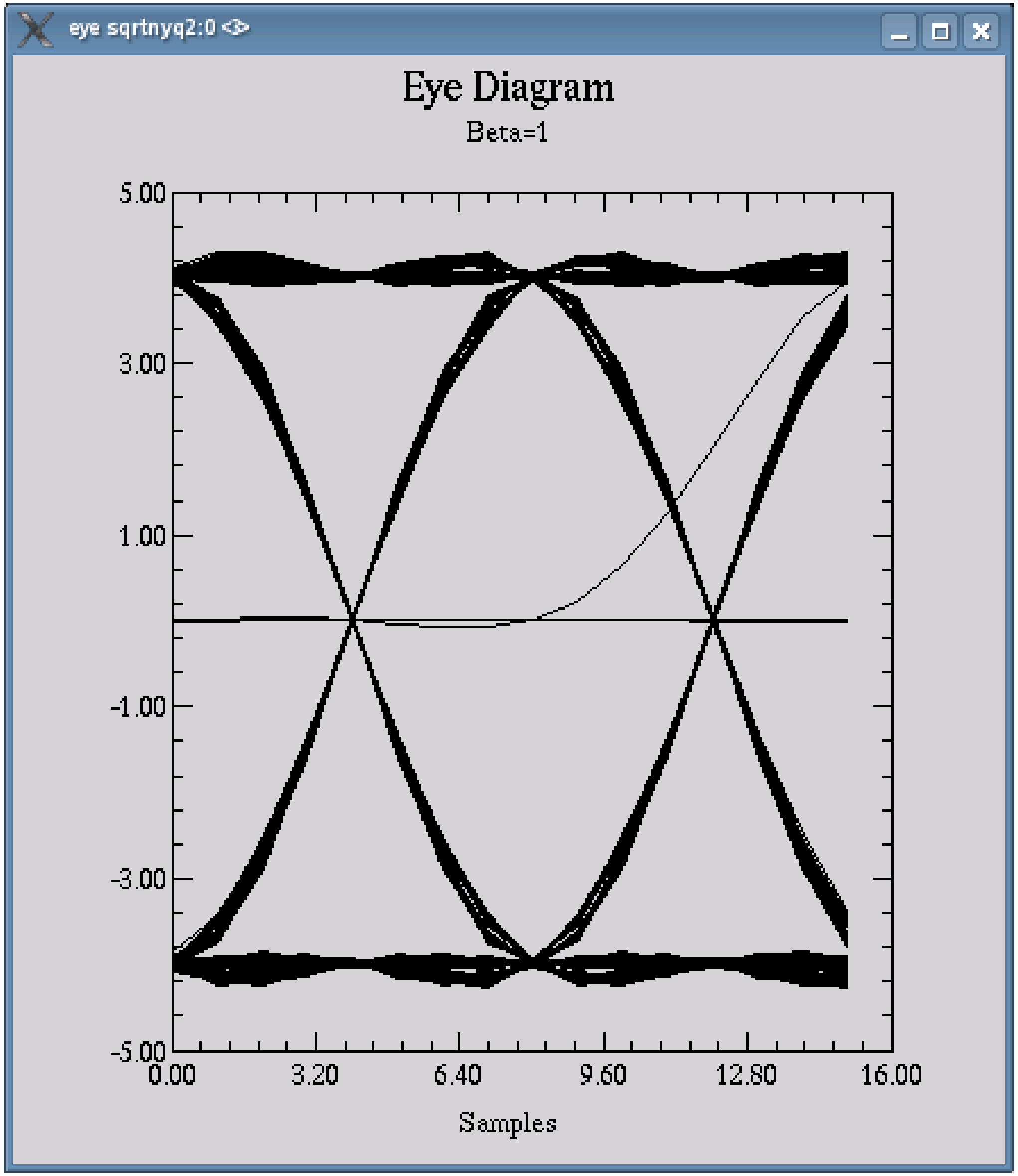
$$R_s = \frac{1}{T}$$

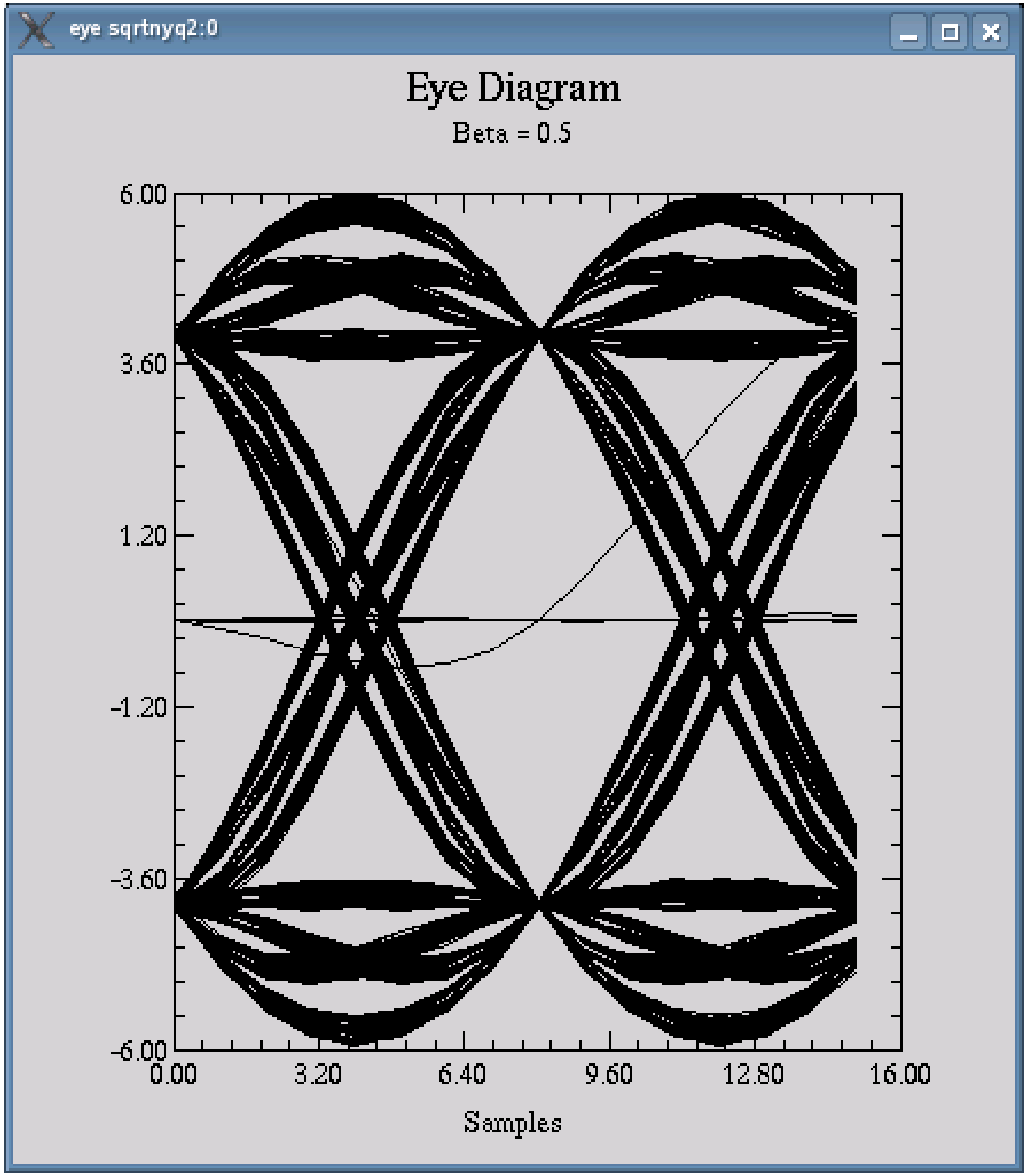



t
t

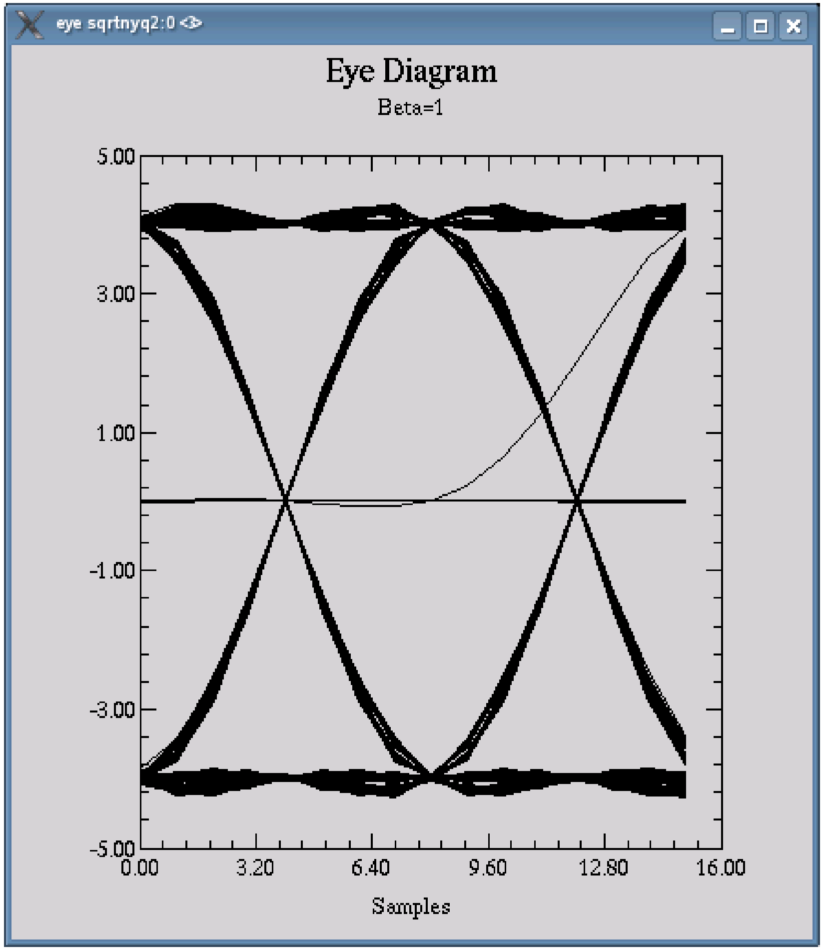


Spectrum Time Domain eye0:0
Nyquist Pulse Shaping Beta=1
5.00
3.00
1.00
-1.00
-3.00
-5.00
1000.00
1080.00
1160.00
1240.00
1320.00
1400.00
Time, Samples

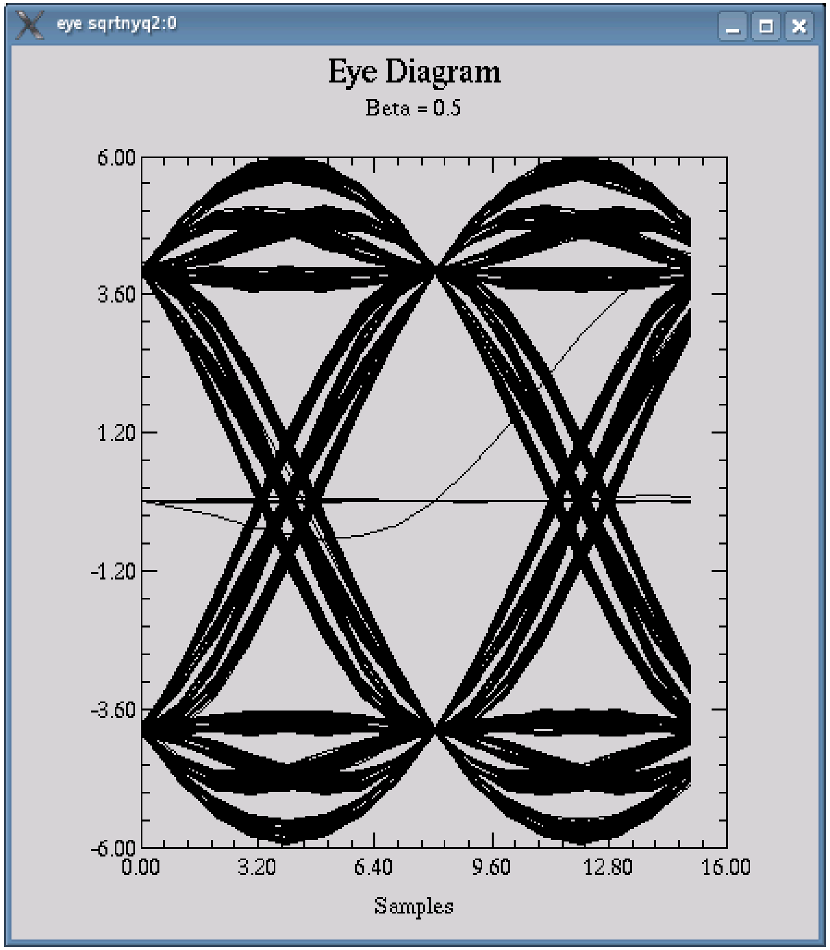

eye sqrtnyq2:0 <3>
Eye Diagram
Beta=1
5.00
3.00
1.00
-1.00
-3.00
-5.00
0.00
3.20
6.40
9.60
12.80
16.00
Samples


eye sqrtnyq2:0
Eye Diagram
Beta = 0.5
6.00
3.60
1.20
-1.20
-3.60
-6.00
0.00
3.20
6.40
9.60
12.80
16.00
Samples

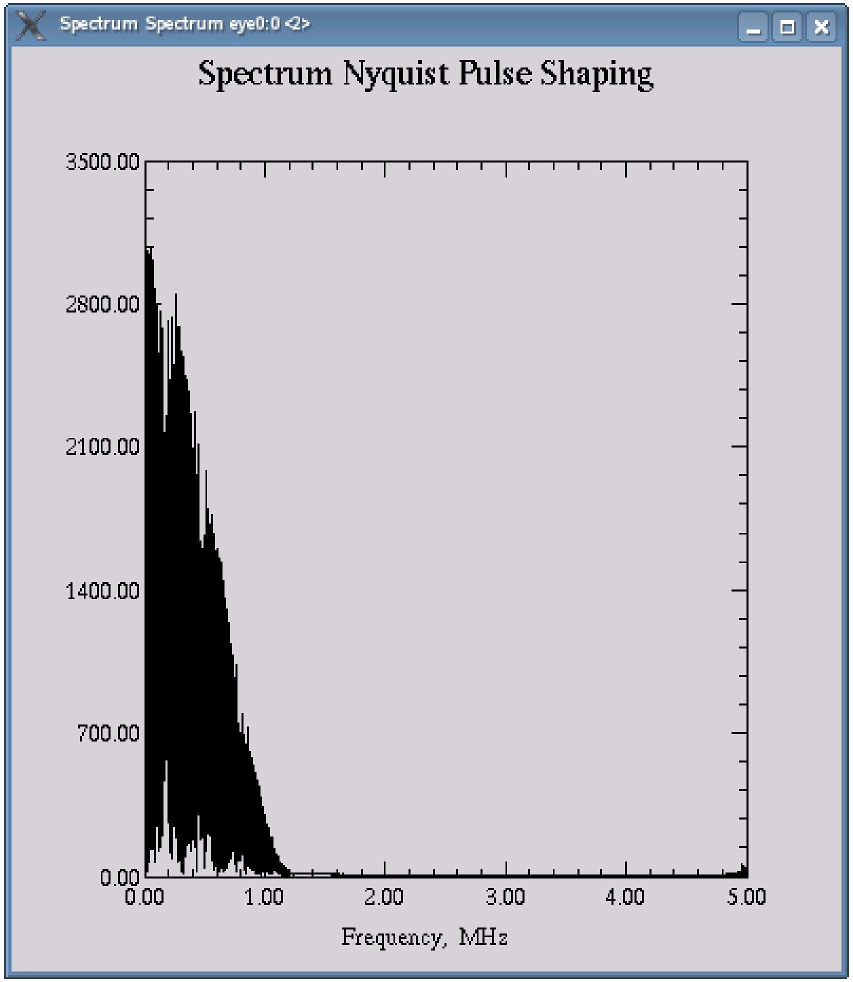
$$\beta = 1.0$$



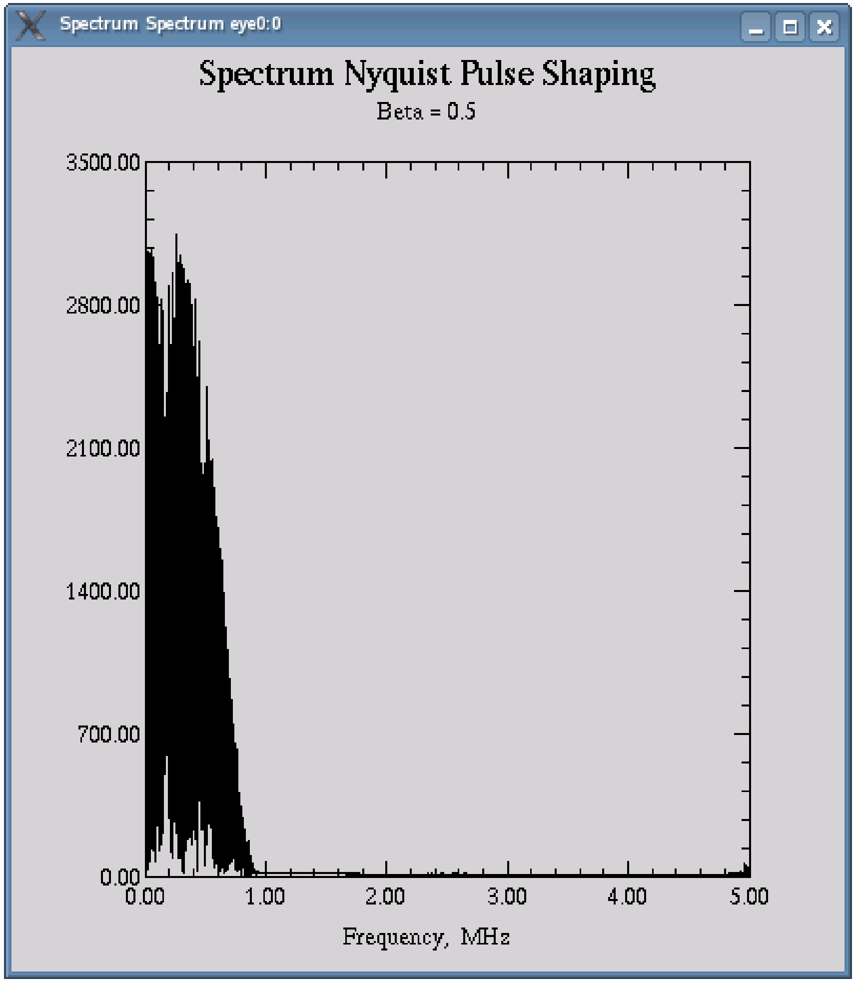
$$\beta = 0.5$$



100% Excess Bandwidth



50% Excess Bandwidth



$\beta = 1.0$ W=1.25 MHz

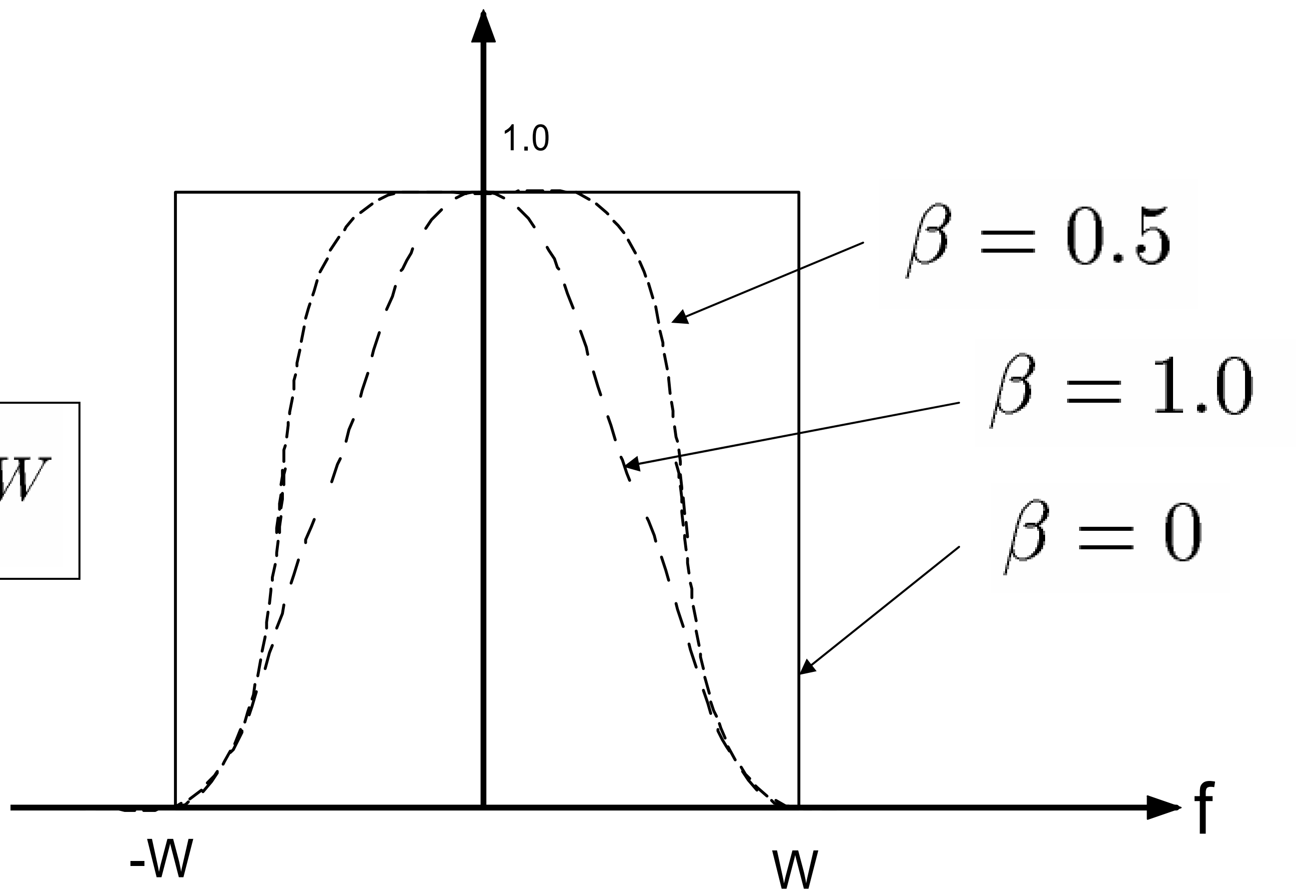
$\beta = 0.5$ W=0.9375 MHz

$$W = \frac{1}{2}(1 + \beta)R_s$$

$$R_s = 1.25 \text{ MSymbols/s}$$

Rolloff Factor and Symbol Rate

$$R_s = \frac{2}{1+\beta}W$$



IEEE 802.16a Single Carrier

8.3.1.5 Baseband Pulse Shaping

Prior to modulation, I and Q signals shall be filtered by square-root raised cosine. A roll-off factor of $\alpha = 0.25$ shall be supported; 0.15 and 0.18 are optional, but defined settings. The ideal square-root cosine is defined in the frequency domain by the transfer function

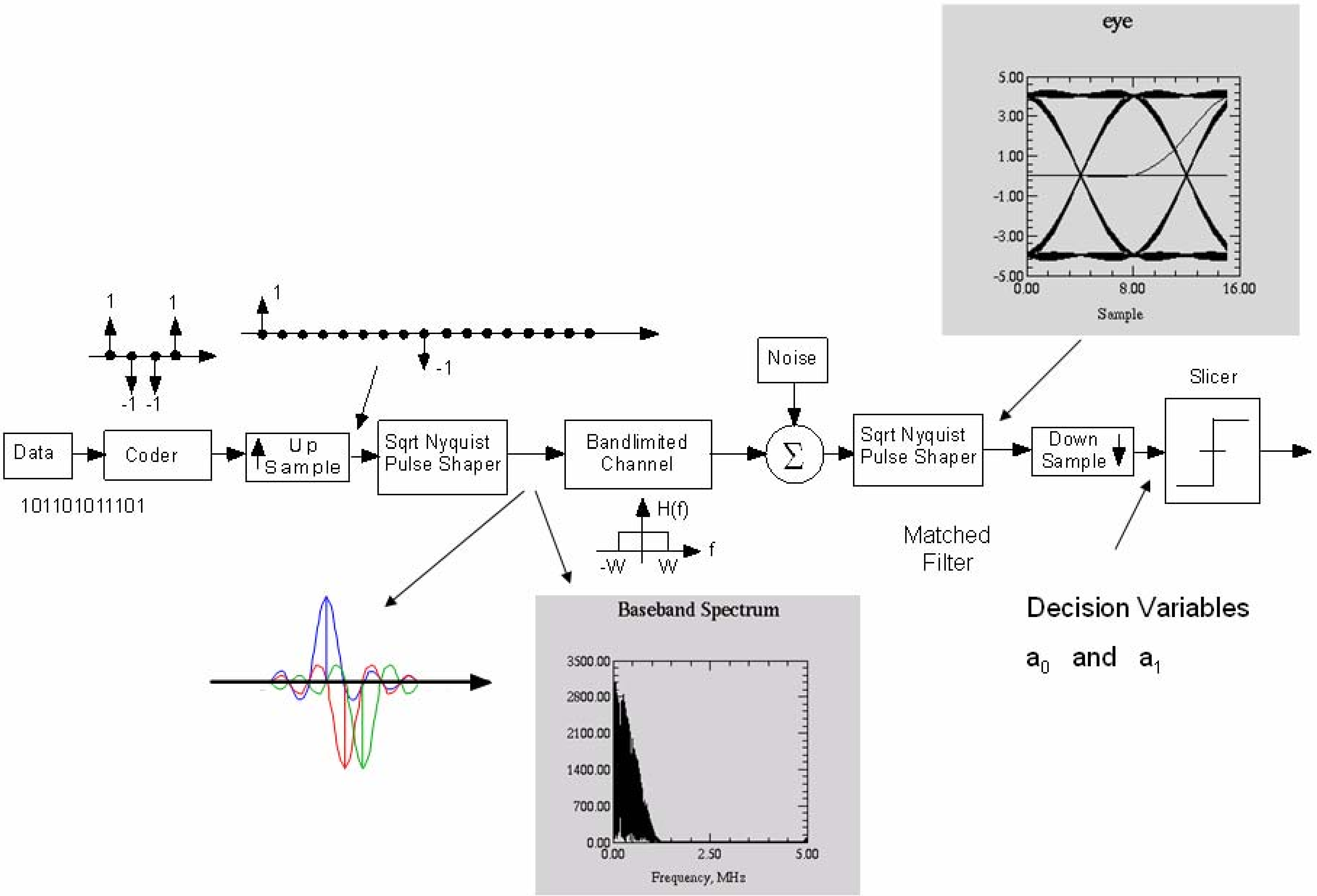
$$H(f) = \begin{cases} 1 & |f| < f_N(1-\alpha) \\ \sqrt{\frac{1}{2} + \frac{1}{2}\sin\left(\frac{\pi}{2f_N}\left[\frac{f_N - |f|}{\alpha}\right]\right)} & f_N(1-\alpha) \le |f| \le f_N(1+\alpha) \\ 0 & |f| \ge f_N(1+\alpha) \end{cases} \qquad (17)$$

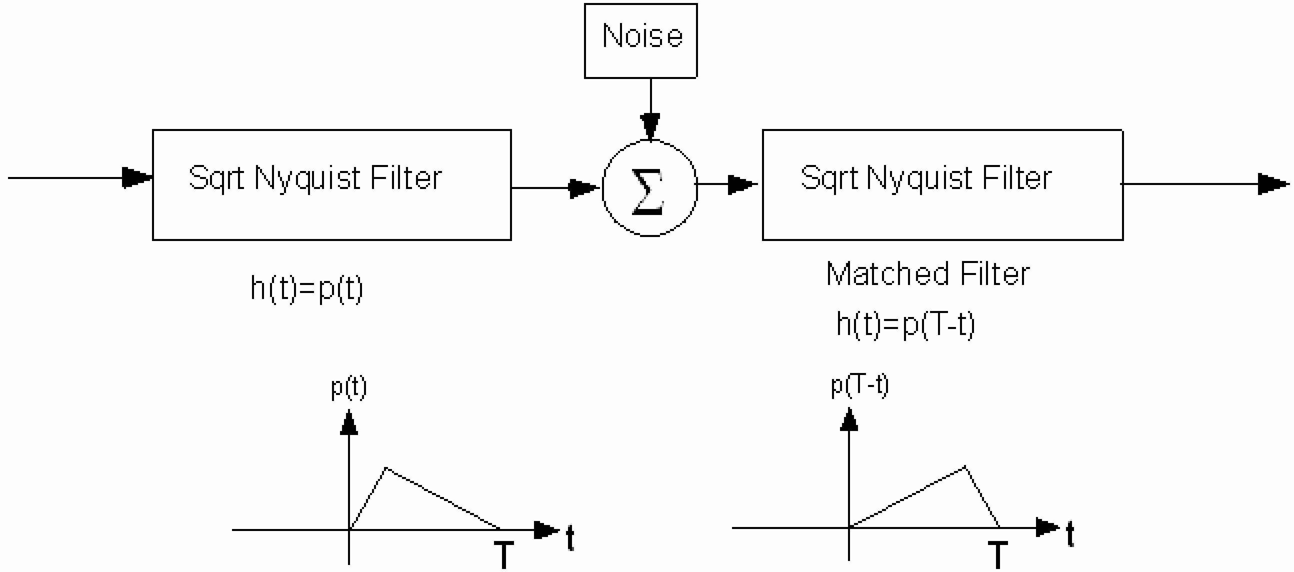
where

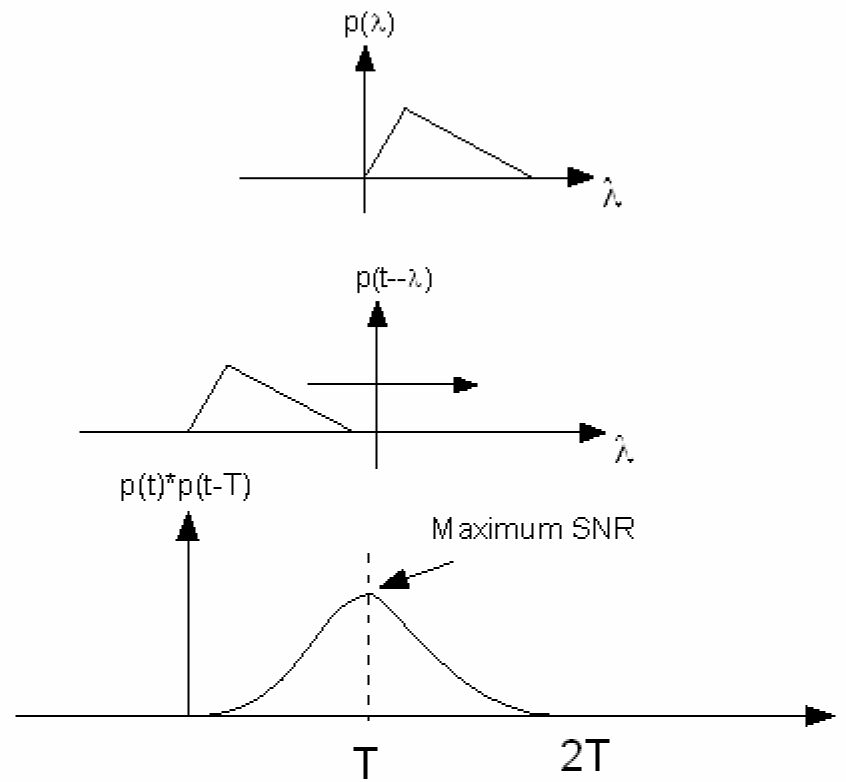
$$f_N = \frac{1}{2T_S} = \frac{R_S}{2}, \qquad (18)$$

f_N is the Nyquist frequency, T_s is the modulation symbol duration, and R_s is the symbol rate.

Baseband Digital Communication Link







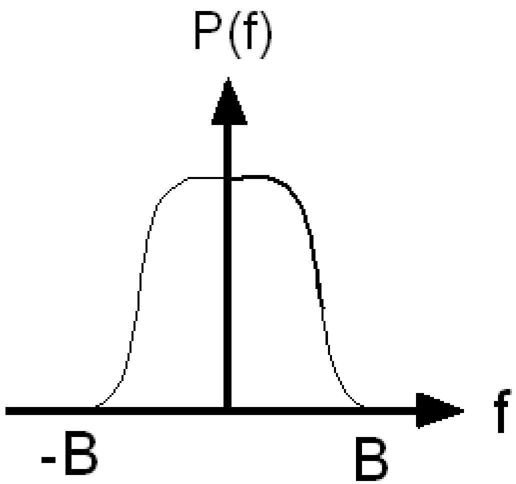
Convolution

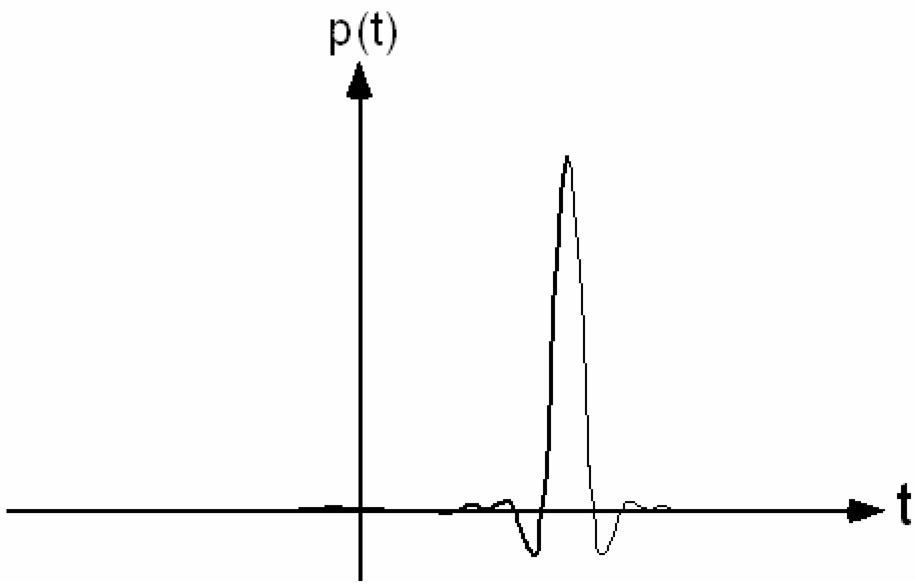
$$z(t) = y(t) * x(t) = \int_{-\infty}^{\infty} y(\lambda)x(t-\lambda)d\lambda$$

$$A\int_{-\infty}^{\infty} |P(f)|^2 df$$

$$A\int_{-\infty}^{\infty} |p(\tau)|^2 d\tau$$

Signal Energy

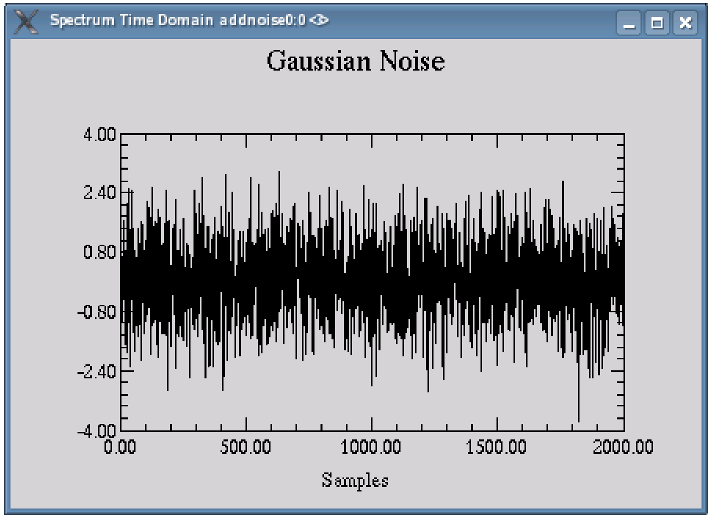




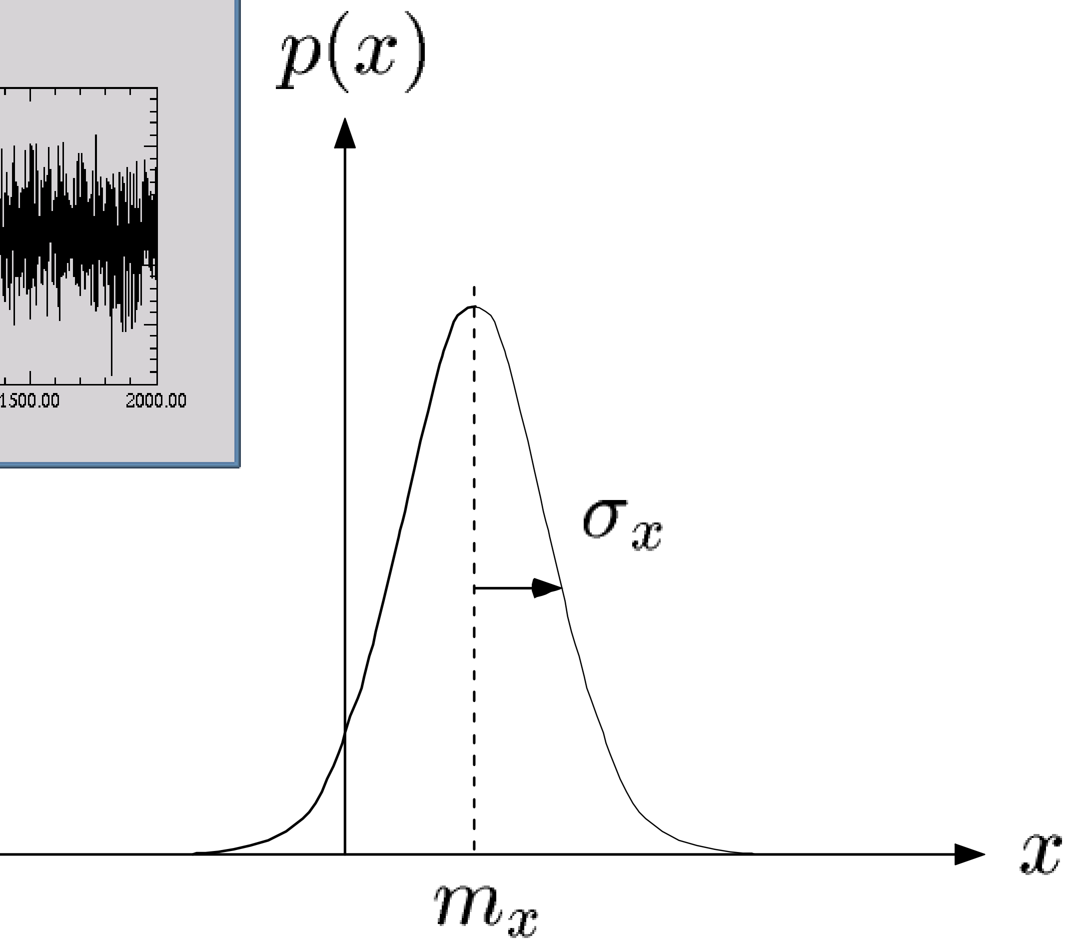
$$Ap(t)$$

$$E_b = A^2 \int_{-\infty}^{\infty} |P(f)|^2 df$$

$$E_b = A^2 \int_{-\infty}^{\infty} |p(\tau)|^2 d\tau$$

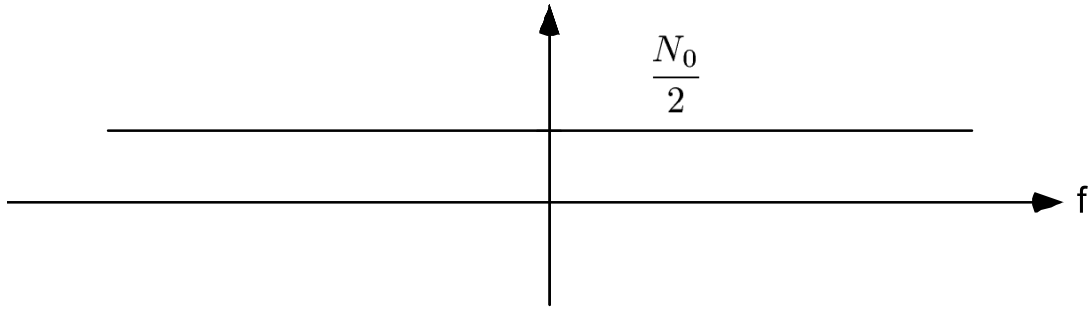


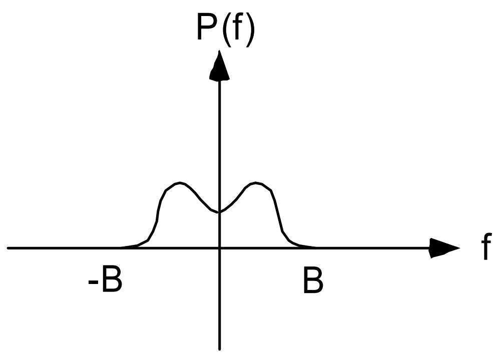
$\sigma_x = 1.0$

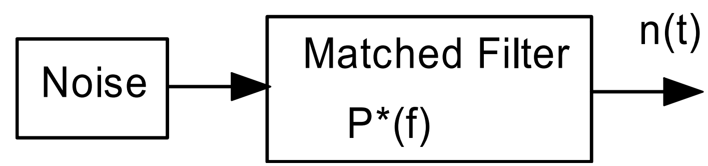


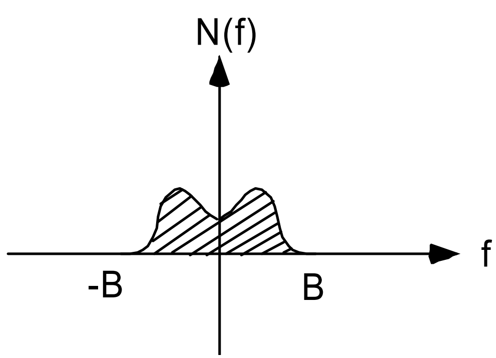
$$p(x) = \frac{1}{\sigma_x\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-m_x}{\sigma_x}\right)^2}$$

Noise Spectral Density

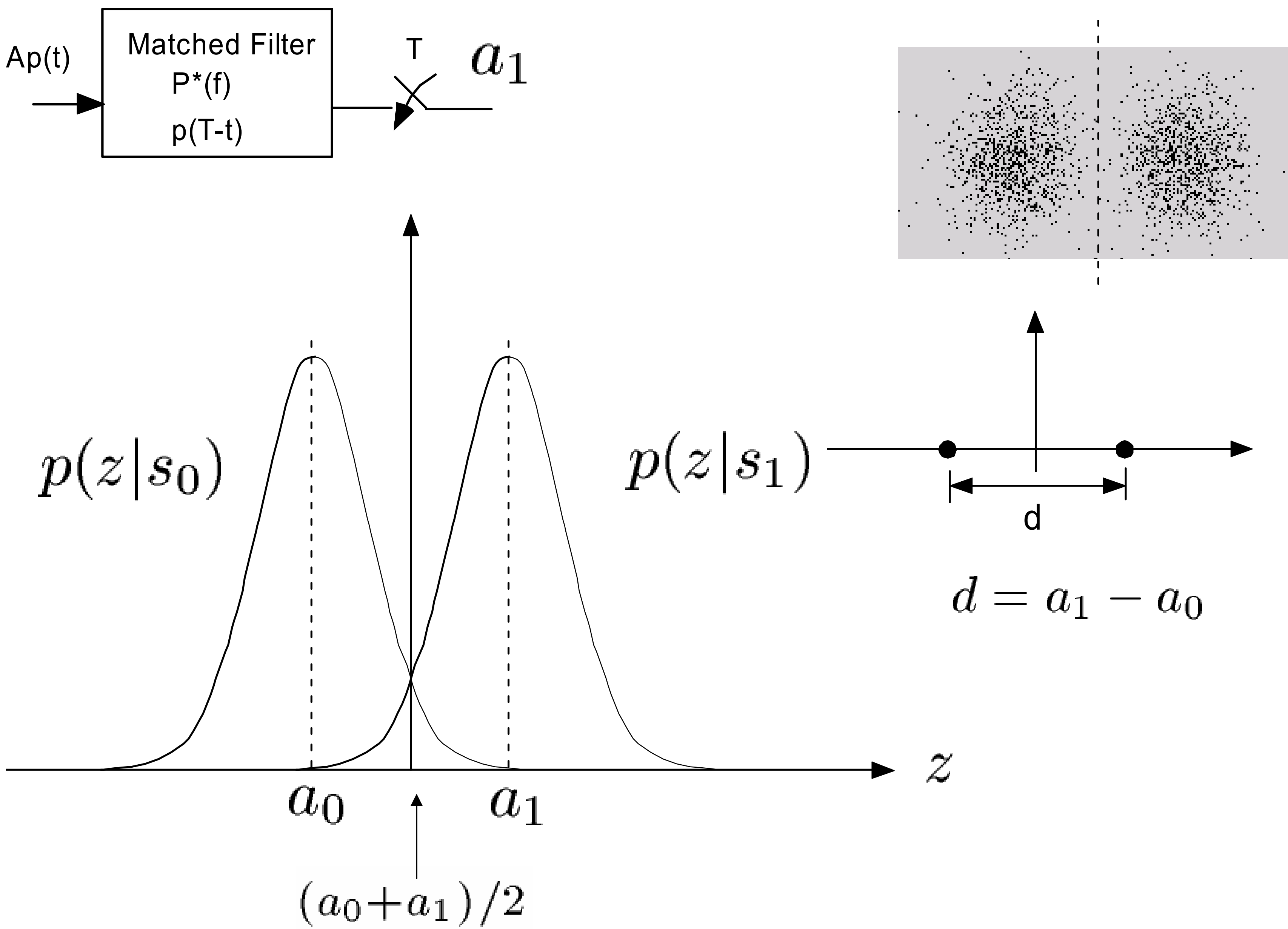




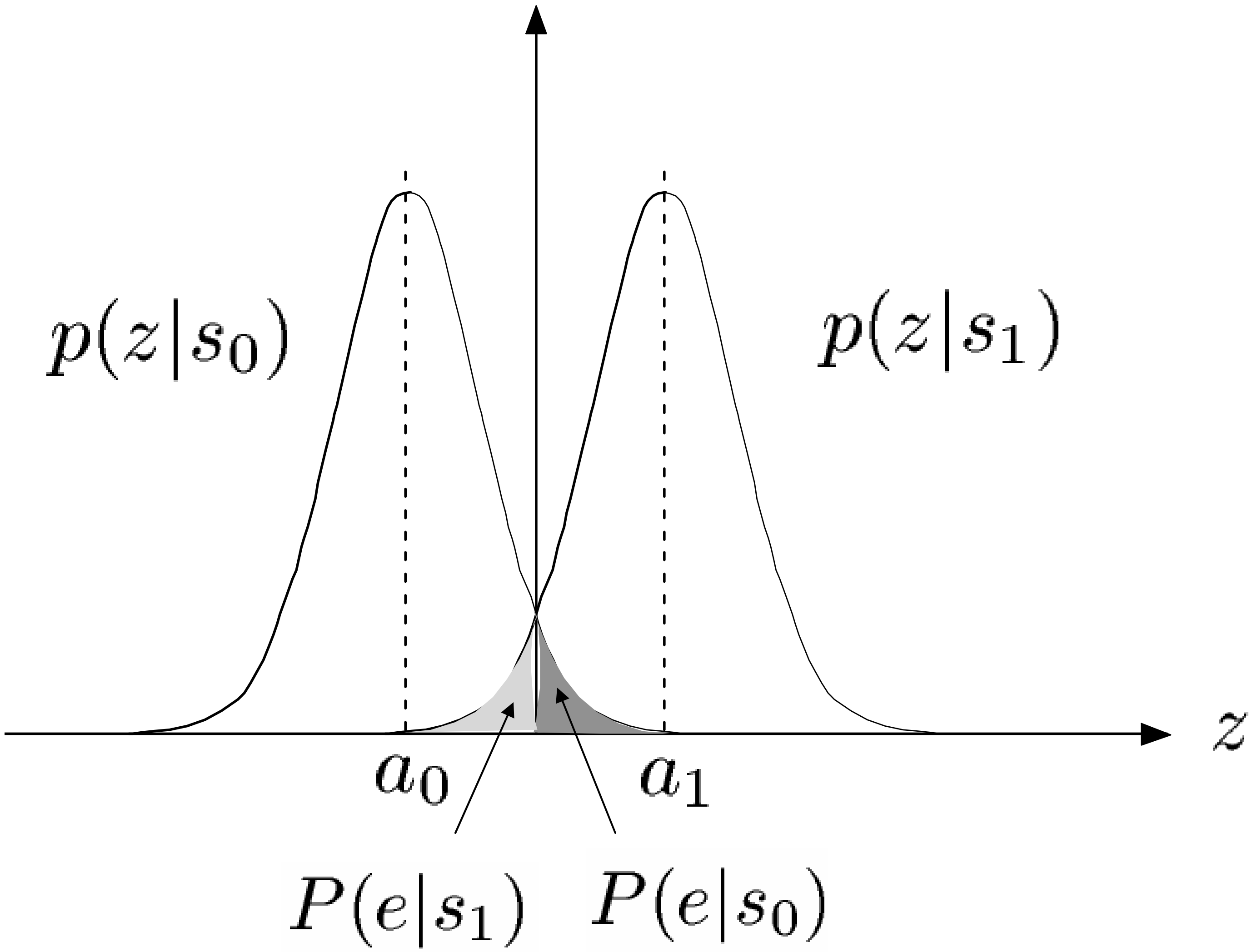




$$\sigma_n^2 = \frac{N_0}{2} \int_{-\infty}^{\infty} |P(f)|^2 df$$



Ap(t)
Matched Filter
P*(f)
p(T-t)
T
a_1
$p(z|s_0)$
$p(z|s_1)$
d
$d = a_1 - a_0$
z
a_0
a_1
$(a_0+a_1)/2$



$p(z|s_0)$
$p(z|s_1)$
z
a_0
a_1
$P(e|s_1)$
$P(e|s_0)$

$$P_e = P(e|s_0)p(s_0) + P(e|s_1)p(s_1)$$

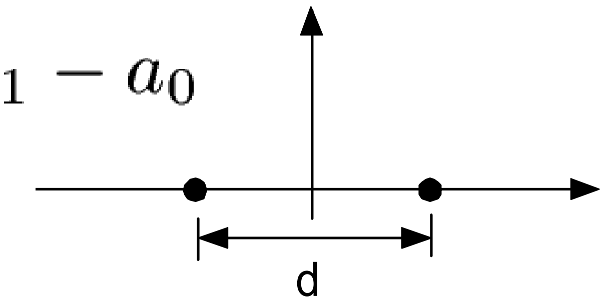
$$p(s_0) = p(s_1) = \frac{1}{2}$$

$$P_e = P(e|s_0) = P(e|s_1)$$

$$p(z|s_0) = \frac{1}{\sigma_n\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{z-a_0}{\sigma_n}\right)^2}$$

$$P_e = \int_{(a_0+a_1)/2}^{\infty} \frac{1}{\sigma_n\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{z-a_0}{\sigma_n}\right)^2} dz$$

$$d = a_1 - a_0$$



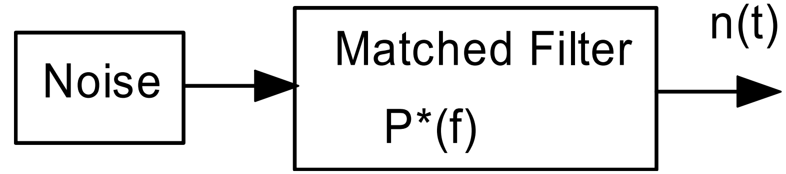
$$Q(x) = \frac{1}{2\pi} \int_x^\infty e^{-\frac{u^2}{2}} du$$

$$x > 3$$

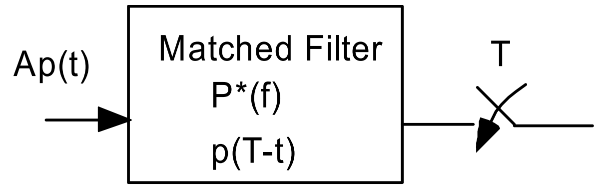
$$Q(x) \approx \frac{1}{x\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

$$P_e = Q\left(\frac{a_1 - a_0}{2\sigma_n}\right)$$

$$\sigma_n^2 = \frac{N_0}{2}\int_{-\infty}^{\infty}|P(f)|^2 df$$



$$a_0 = \; -A\int_{-\infty}^{\infty}|P(f)|^2 df$$



$$A\int_{-\infty}^{\infty}|P(f)|^2 df$$

$$A\int_{-0}^{T}|p(\tau)|^2 d\tau$$

$$a_1 = \; A\int_{-\infty}^{\infty}|P(f)|^2 df$$

$$\frac{(a_1 - a_0)^2}{4\sigma_n^2} = \frac{4A^2(\int_{-\infty}^{\infty}|P(f)|^2 df)^2}{4\frac{N_0}{2}\int_{-\infty}^{\infty}|P(f)|^2 df}$$

Received Signal $Ap(t)$

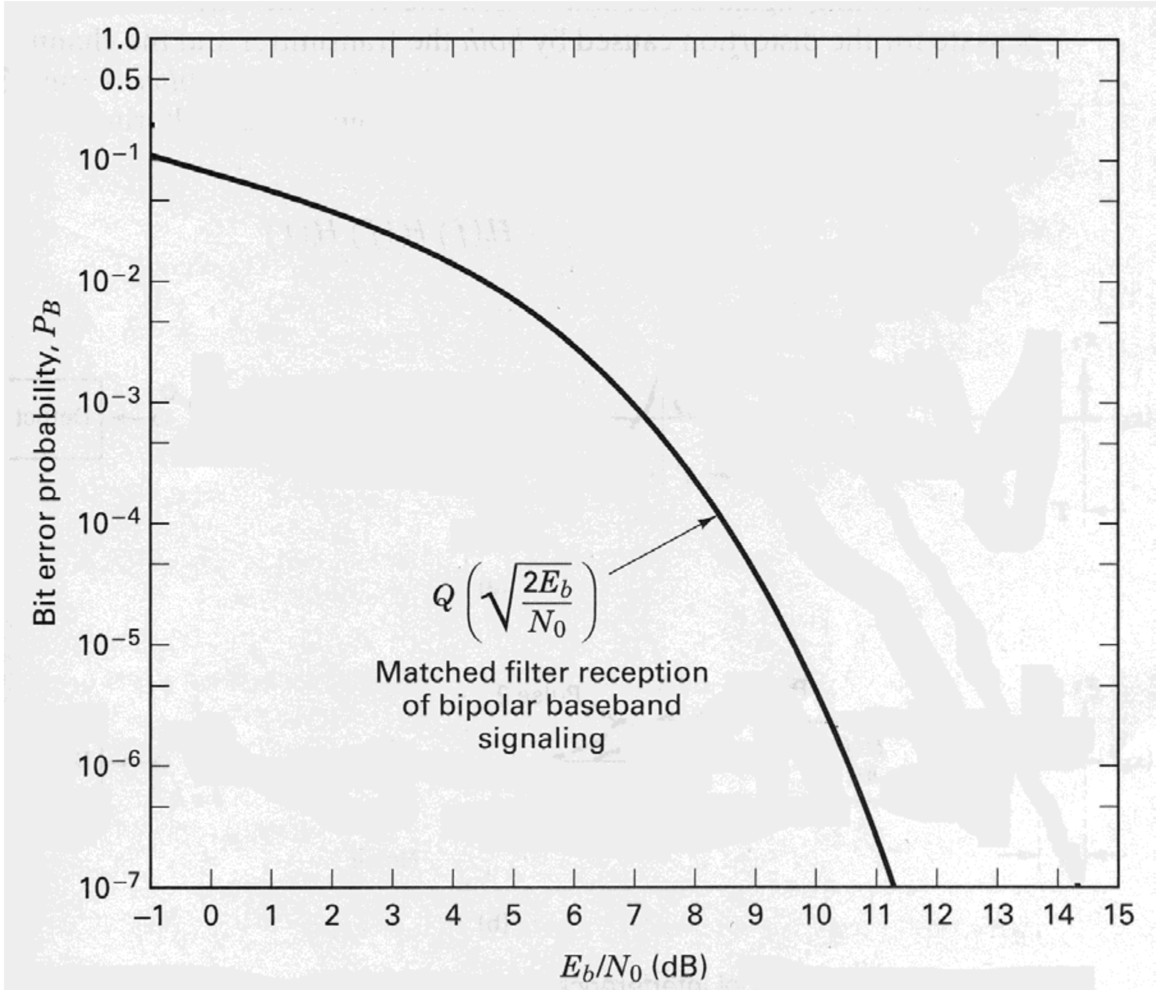
Energy

$$E_b = A^2\int_{-\infty}^{\infty}|P(f)|^2 df$$

$$\frac{(a_1 - a_0)^2}{4\sigma_n^2} = \frac{2A^2\int_{-\infty}^{\infty}|P(f)|^2 df}{N_0}$$

$$\frac{(a_1 - a_0)^2}{4\sigma_n^2} = \frac{2E_b}{N_0} \longrightarrow \frac{(a_1 - a_0)}{2\sigma_n} = \sqrt{\frac{2E_b}{N_0}}$$

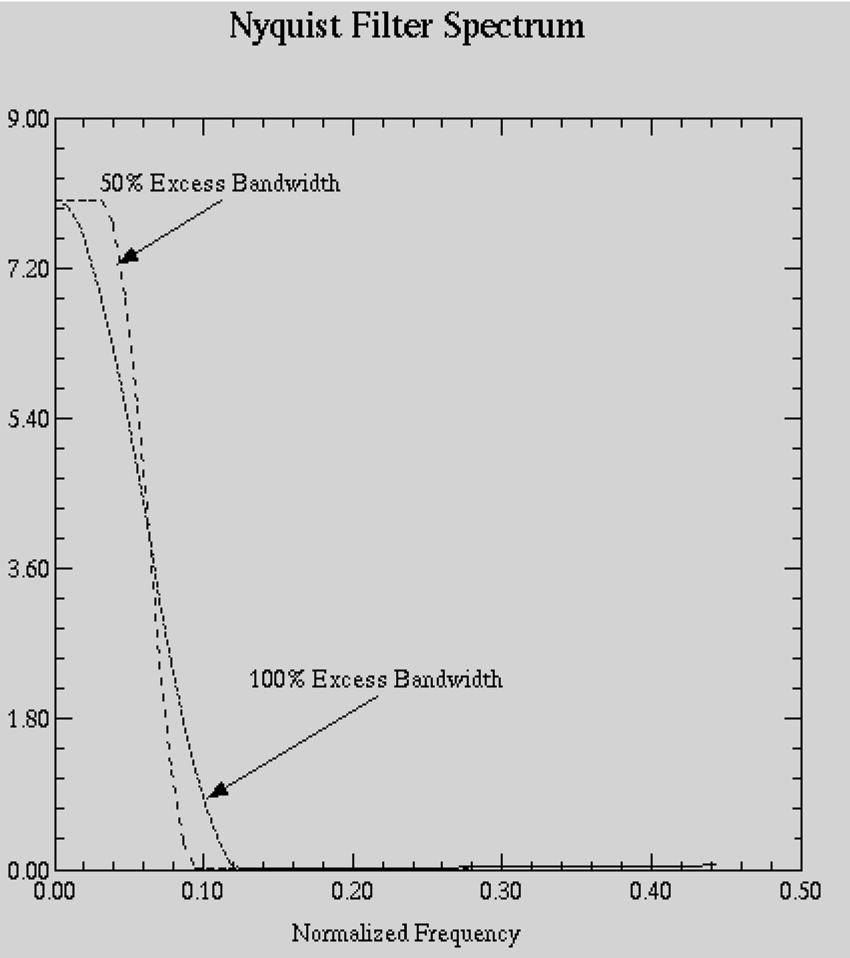
$$P_e = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

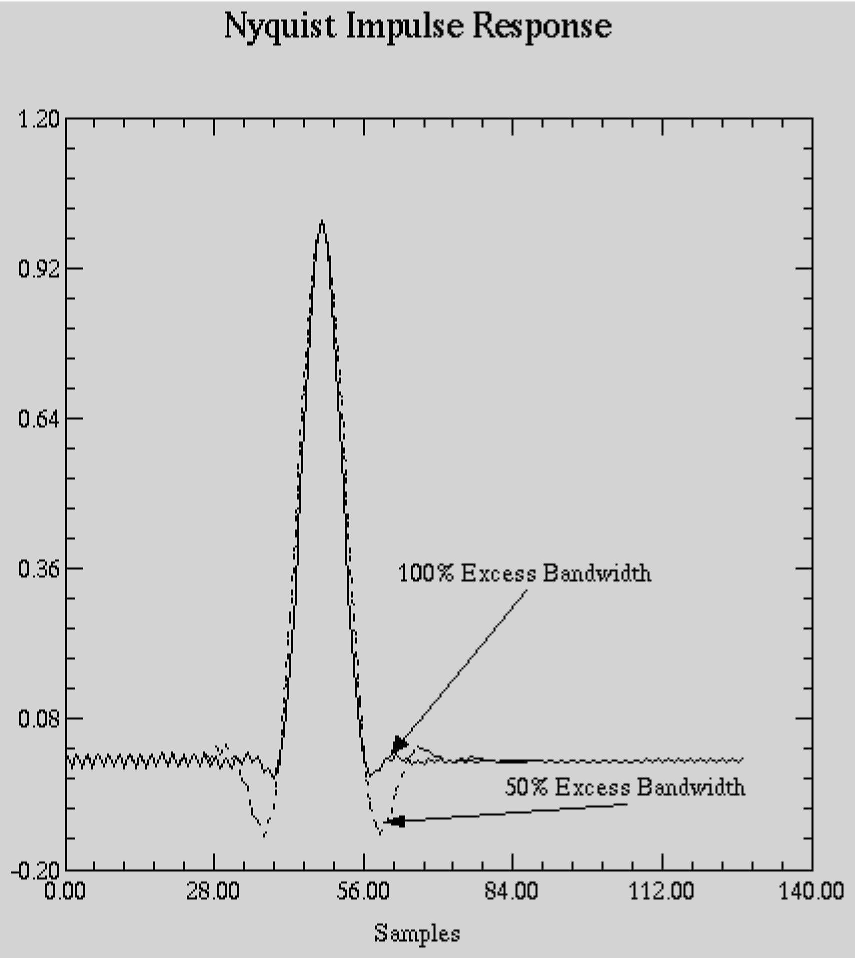


Source:

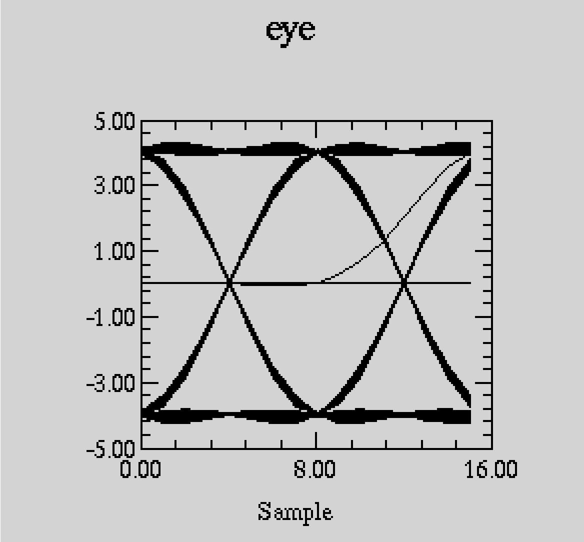
Bernard Sklar, Digital Communications, Prentice Hall, 2001

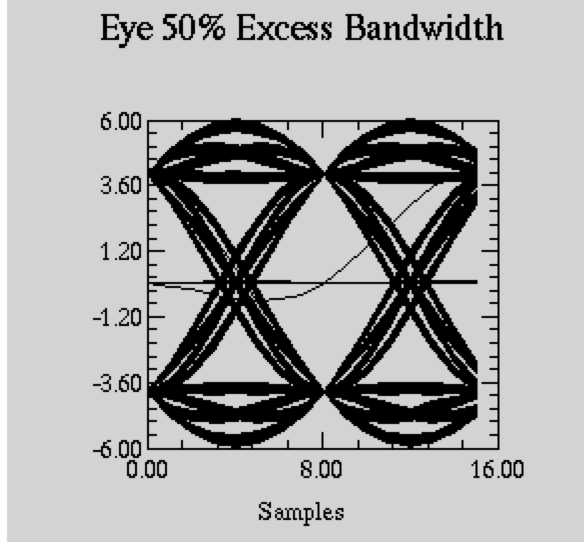
Nyquist Pulse Shaping

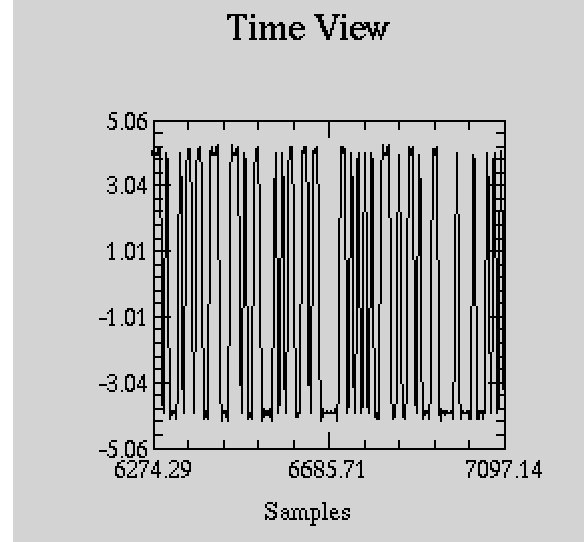


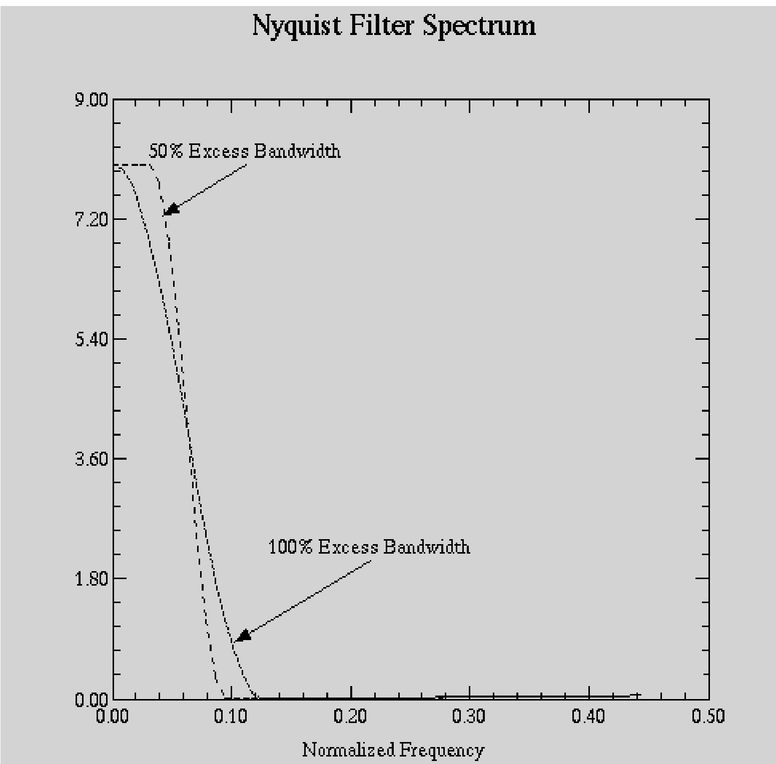


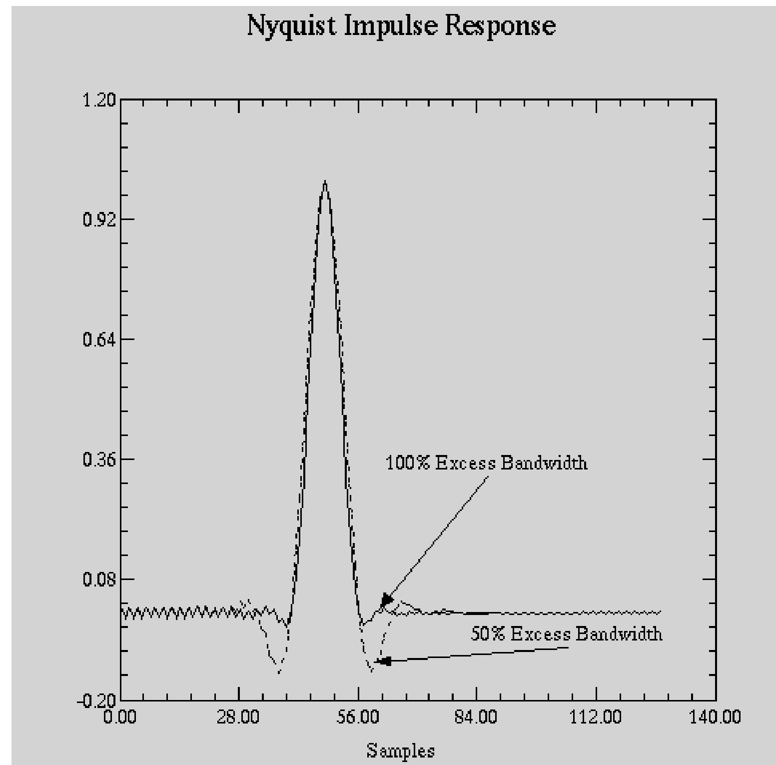
$$h(t) = \frac{\sin \pi t/T}{\pi t/T} \frac{\cos \beta \pi t/T}{1 - 4\beta^2 t^2/T^2}$$


eye
5.00
3.00
1.00
-1.00
-3.00
-5.00
0.00
8.00
16.00
Sample

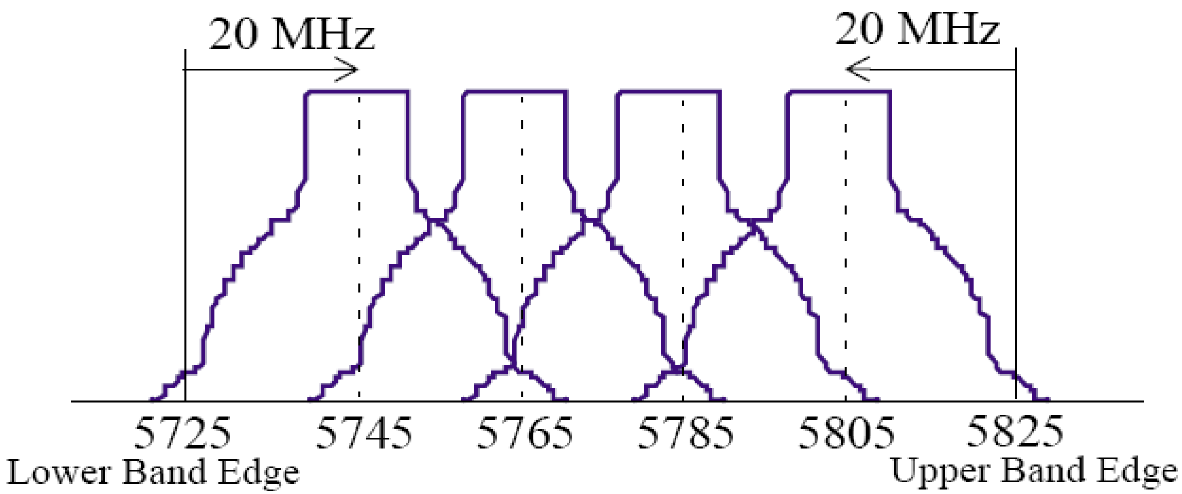

Eye 50% Excess Bandwidth
6.00
3.60
1.20
-1.20
-3.60
-6.00
0.00
8.00
16.00
Samples


Time View
5.06
3.04
1.01
-1.01
-3.04
-5.06
6274.29
6685.71
7097.14
Samples


Nyquist Filter Spectrum
9.00
7.20
5.40
3.60
1.80
0.00
50% Excess Bandwidth
100% Excess Bandwidth
0.00
0.10
0.20
0.30
0.40
0.50
Normalized Frequency


Nyquist Impulse Response
1.20
0.92
0.64
0.36
0.08
-0.20
100% Excess Bandwidth
50% Excess Bandwidth
0.00
28.00
56.00
84.00
112.00
140.00
Samples

Bandpass Channels



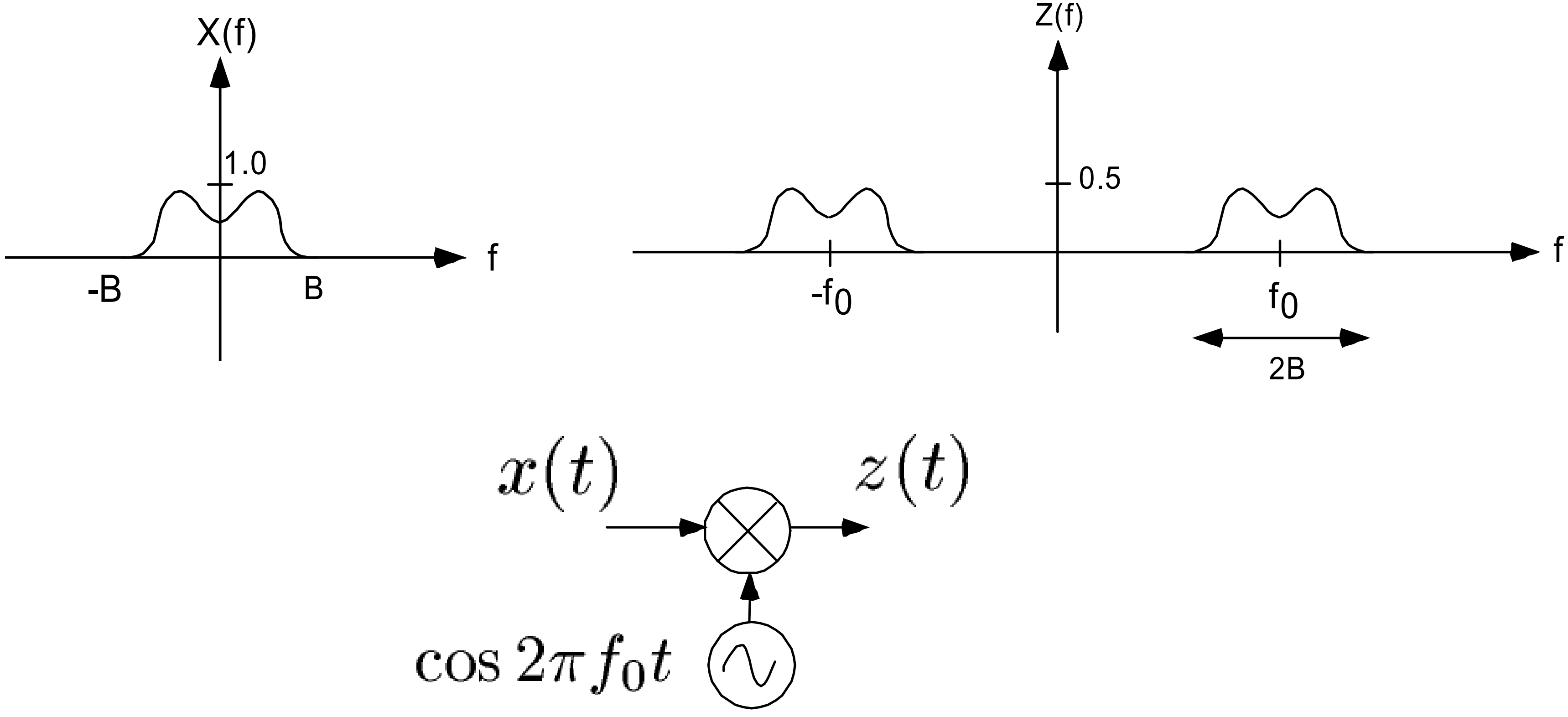
Upper U-NII Bands: 4 Carriers in 100 MHz / 20 MHz Spacing

Modulation

$$x(t)e^{j2\pi f_0 t} \longleftrightarrow X(f - f_0)$$

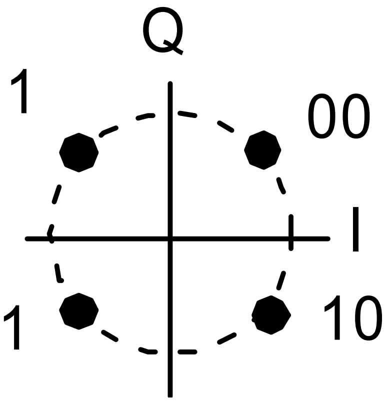
$$\cos 2\pi f_0 t = \frac{1}{2}(e^{j2\pi f_0 t} + e^{-j2\pi f_0 t})$$

$$x(t)\cos 2\pi f_0 t \longleftrightarrow \frac{1}{2}[X(f - f_0) + X(f + f_0)]$$


X(f)
1.0
f
-B
B
Z(f)
0.5
f
$-f_0$
f_0
2B
$x(t)$
$z(t)$
$\cos 2\pi f_0 t$

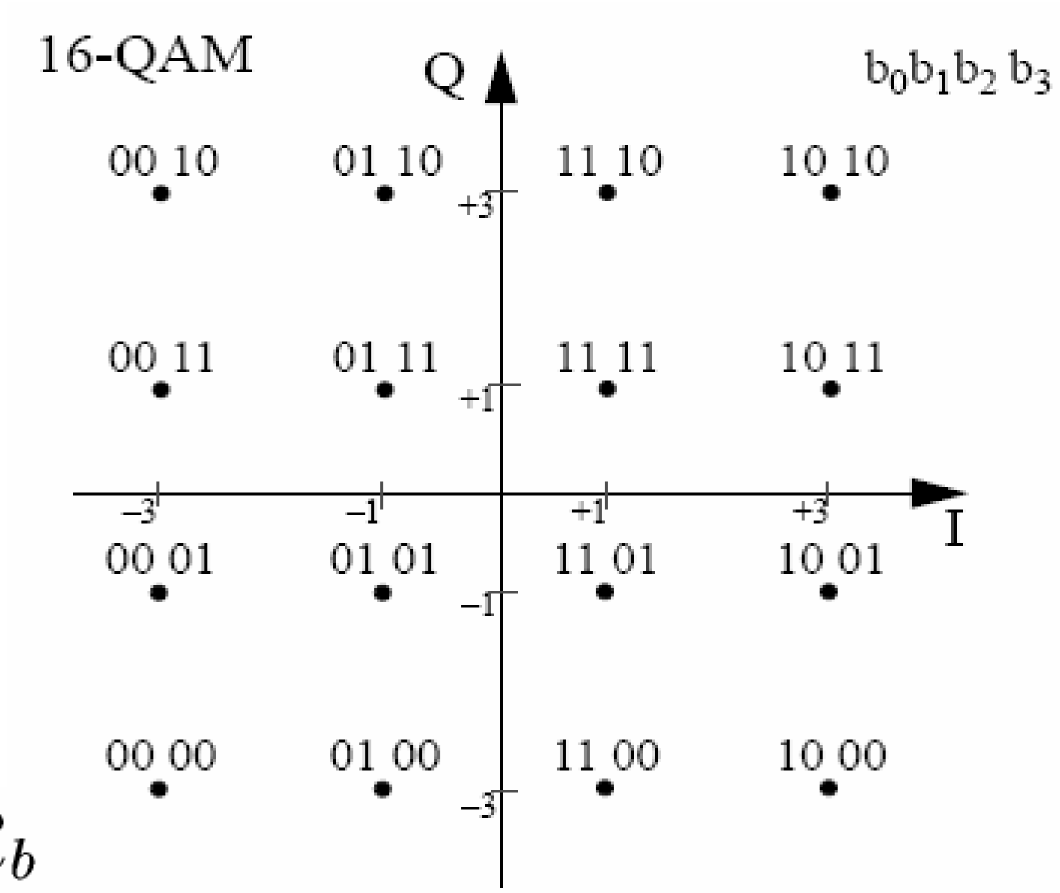
QPSK Constellation

I = -1
Q = 1



$$R_s = \frac{R_b}{2}$$

16 QAM



$$R_s = \frac{R_b}{4}$$

For fixed bandwidth throughput limited by among others:

- SNR (Power Limitation)
- Nonlinearity
- Fading Multipath Channel
- PLL Phase Noise

For example to use 256 QAM (8 bits/symbol) need higher SNR (Greater Power), High linearity in PA and also RF Receive Chain.

This compounded by high peak to average power ratio.

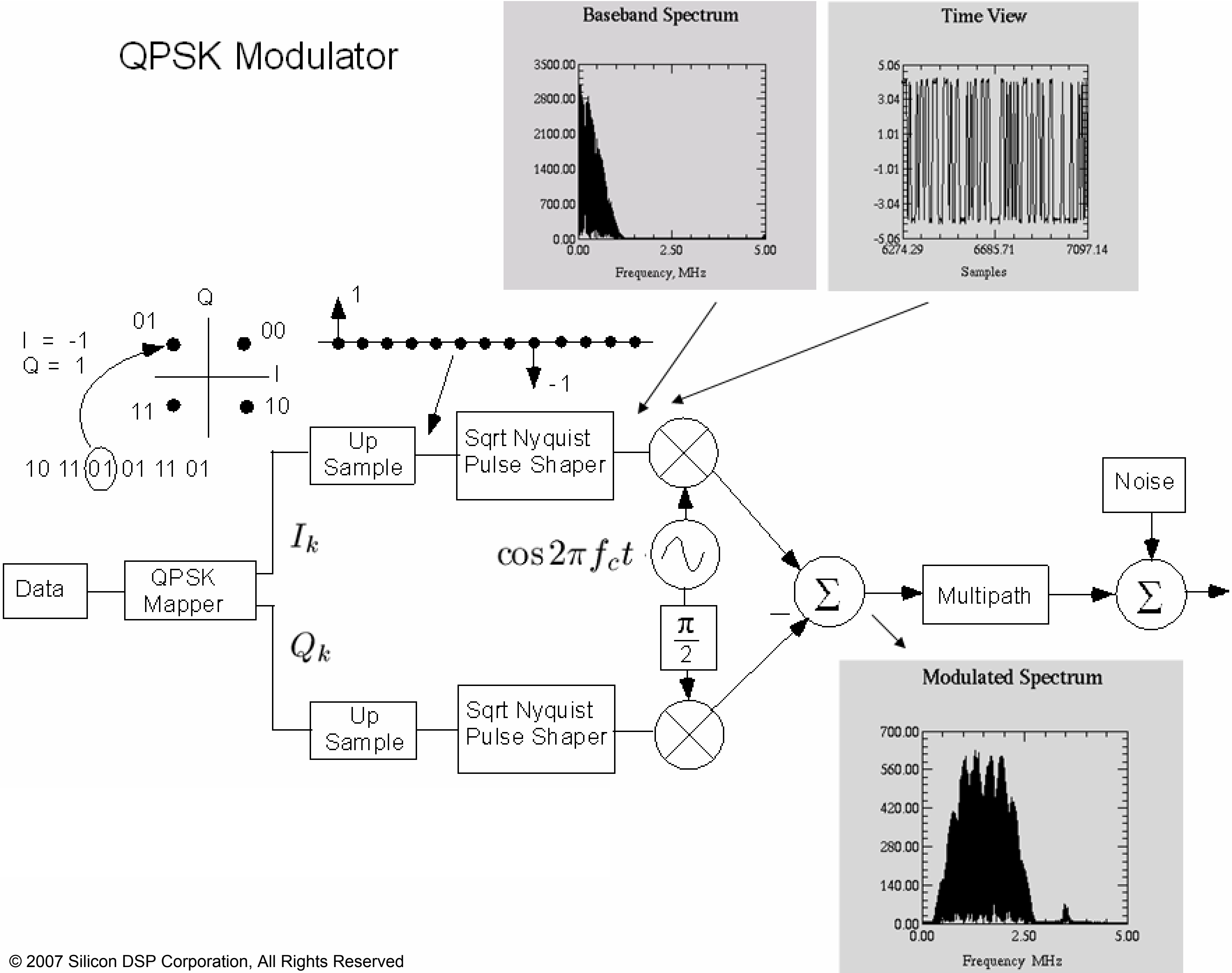
Shannon-Hartley Capacity Theorem

$$C = W \log_2(1 + \frac{S}{N})$$

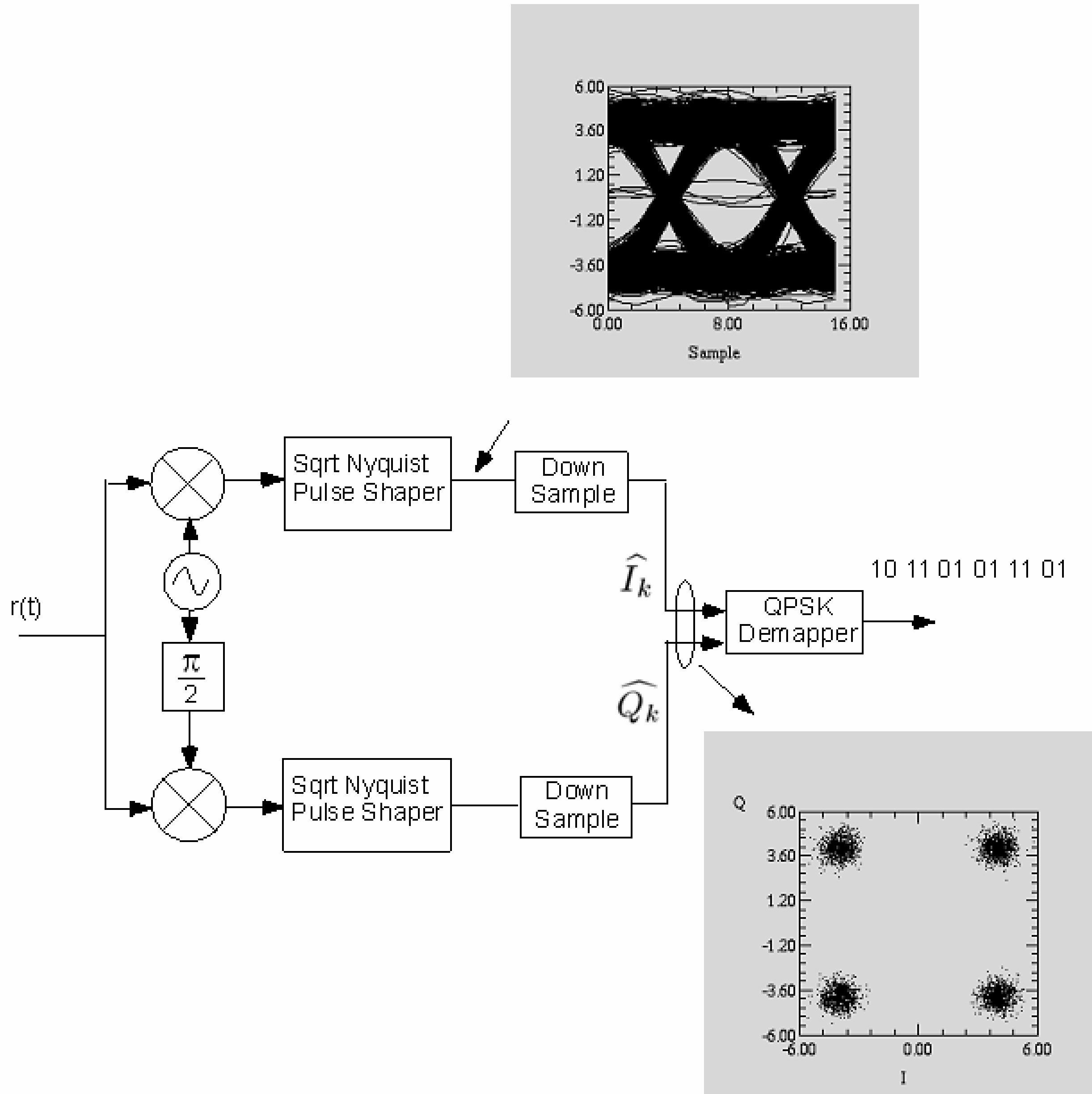
W in Hz

C bits/s

QPSK Modulator

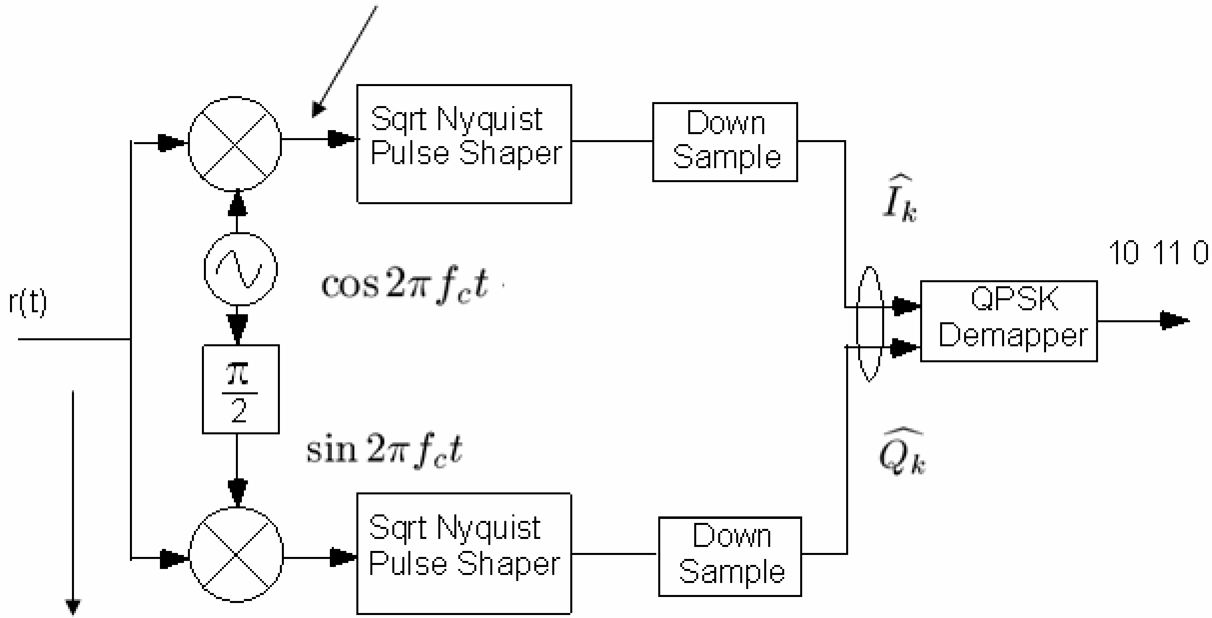


QPSK Demodulator



Quadrature Demodulation

$$r(t)\cos 2\pi f_c t = I_k \cos^2 2\pi f_c t - Q_k \sin 2\pi f_c t \cos 2\pi f_c t$$



$$r(t) = I_k \cos 2\pi f_c t - Q_k \sin 2\pi f_c t$$

$$\cos^2\alpha = \frac{1}{2}(1+\cos 2\alpha)$$

$$\cos\alpha\sin\beta = \frac{1}{2}\sin(\alpha-\beta) + \frac{1}{2}\sin(\alpha+\beta)$$

Baseband

Filtered Out

$$r(t)\cos 2\pi f_c t = \frac{1}{2}I_k + \frac{1}{2}I_k \cos 4\pi f_c t - \frac{1}{2}Q_k \sin 4\pi f_c t$$

$$r(t) = I_k \cos 2\pi f_c t - Q_k \sin 2\pi f_c t$$

$$X_k = I_k + jQ_k$$

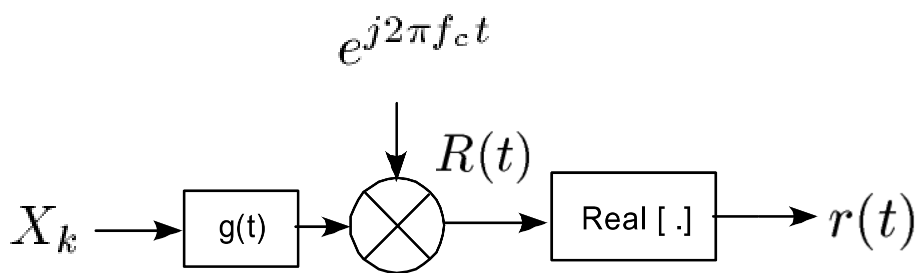
$$e^{j2\pi f_c t} = \cos 2\pi f_c t + j \sin 2\pi f_c t$$

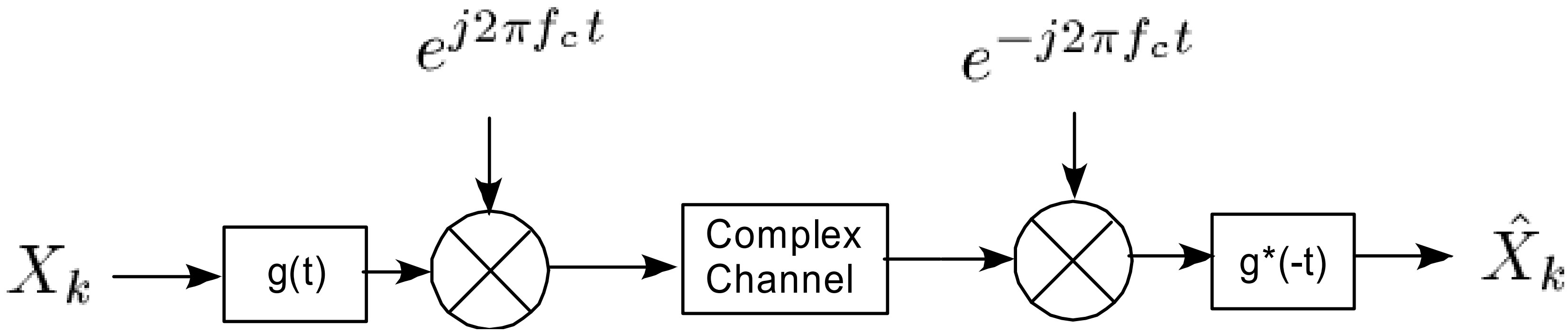
$$R(t) = X_k e^{j2\pi f_c t}$$

Complex Envelope

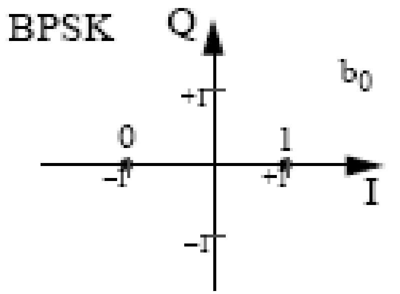
$$R(t) = I_k \cos 2\pi f_c t - Q_k \sin 2\pi f_c t + j(I_k \cos 2\pi f_c t + Q_k \sin 2\pi f_c t)$$

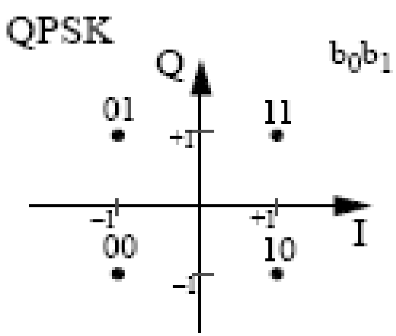
$$r(t) = Real\left[R(t)\right] = I_k \cos 2\pi f_c t - Q_k \sin 2\pi f_c t$$

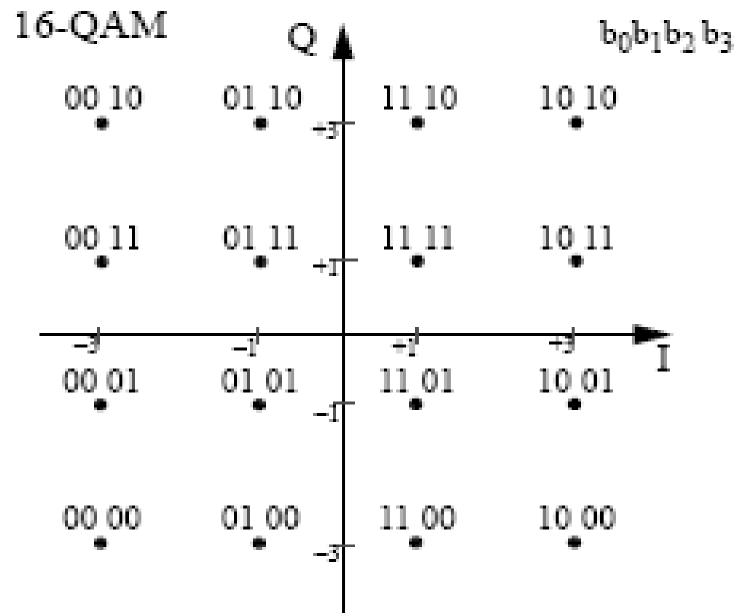



X_k
g(t)
$e^{j2\pi f_c t}$
Complex
Channel
$e^{-j2\pi f_c t}$
g*(-t)
$\hat{X}_k$

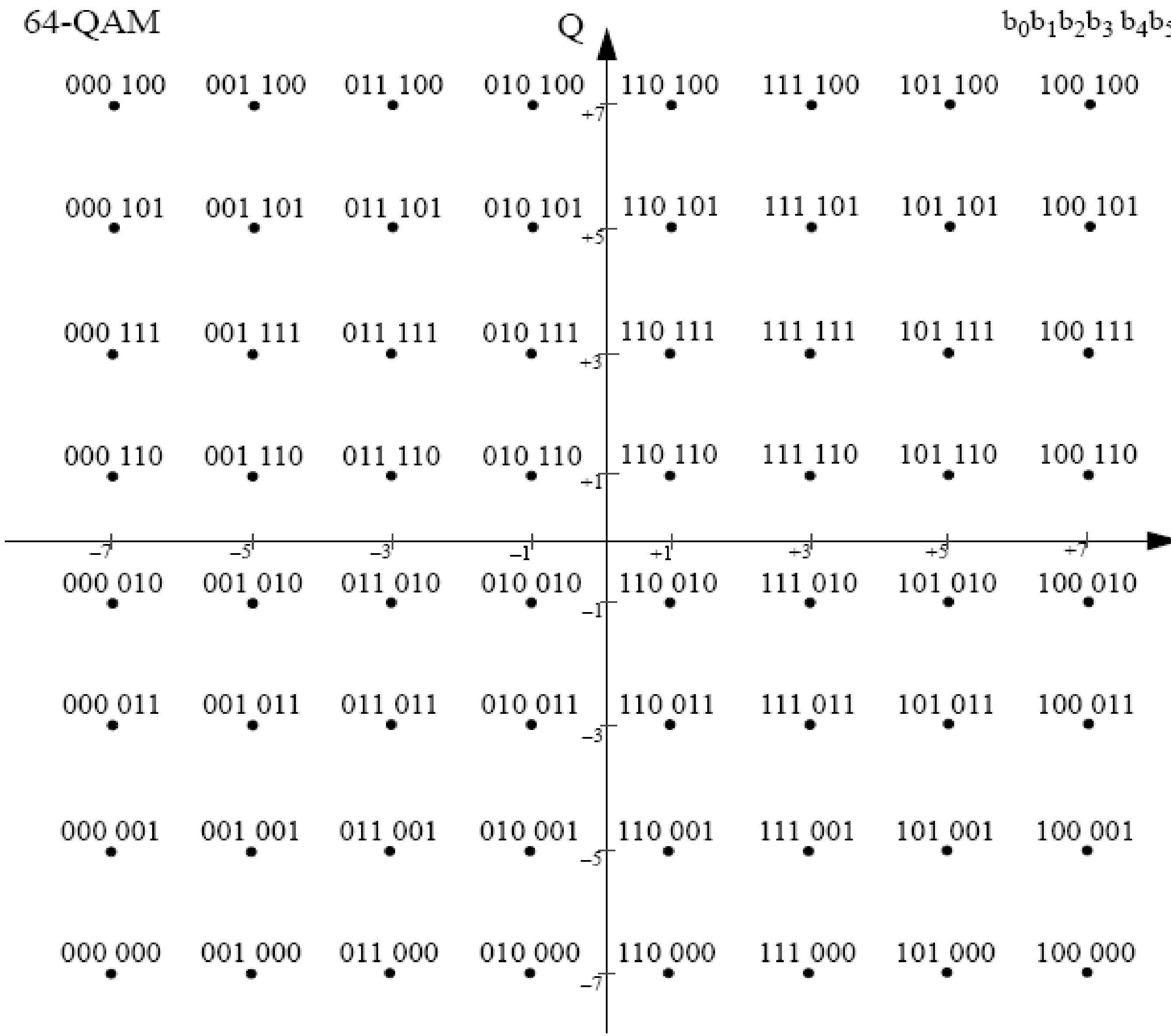
Constellations 802.11a

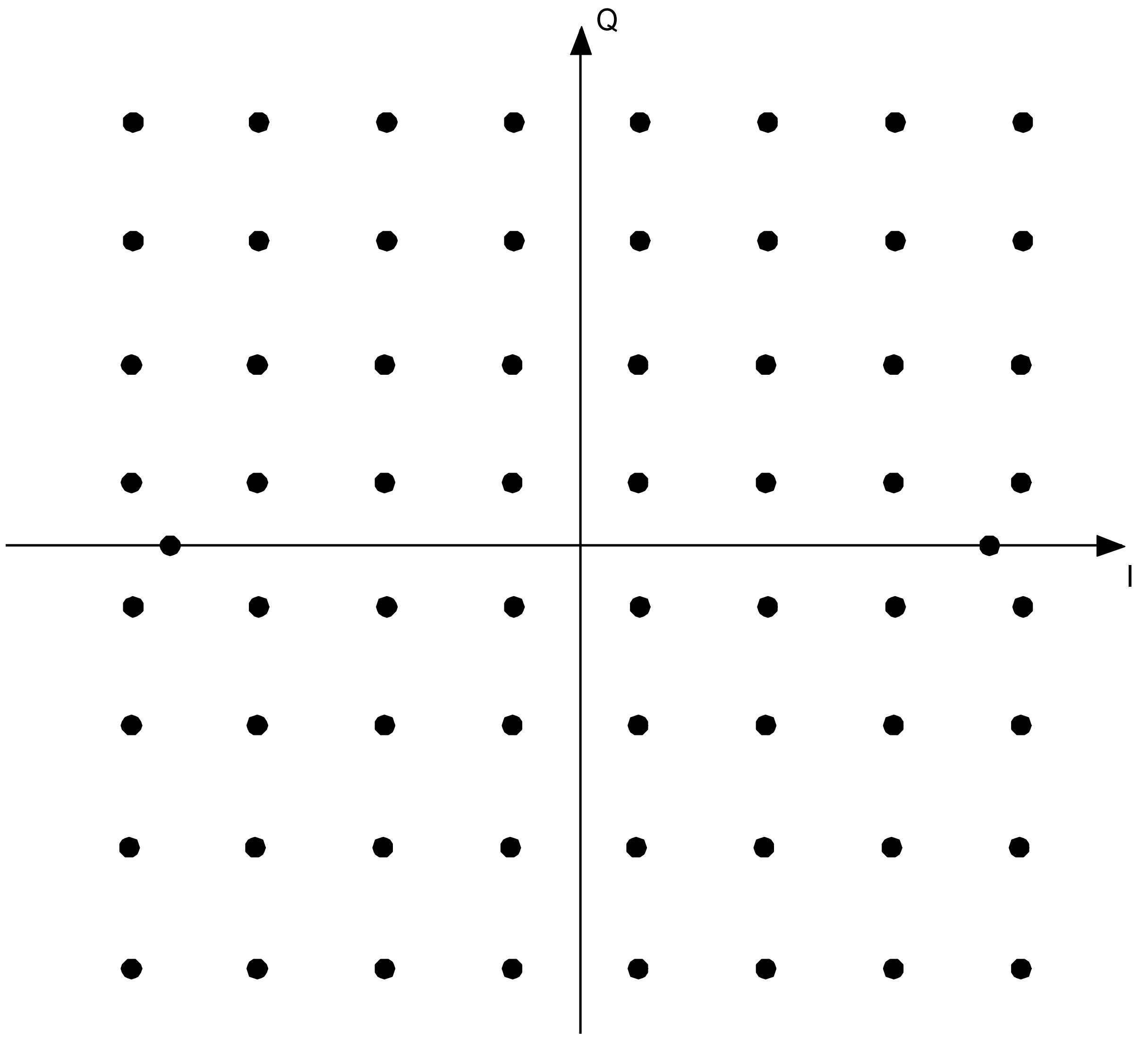


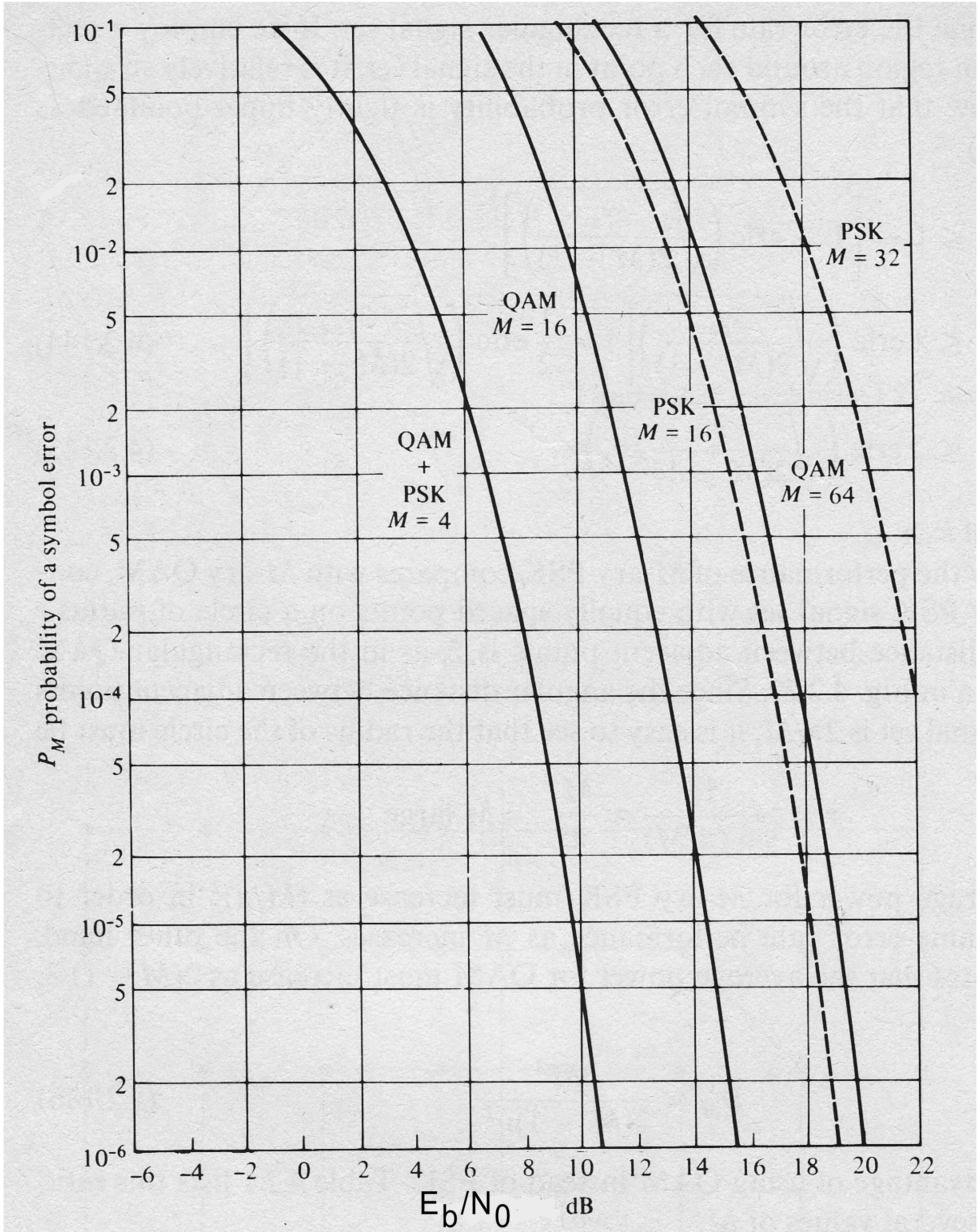




64 QAM


64-QAM
Q
I
b$_0$b$_1$b$_2$b$_3$ b$_4$b$_5$
000 100
001 100
011 100
010 100
110 100
111 100
101 100
100 100
000 101
001 101
011 101
010 101
110 101
111 101
101 101
100 101
000 111
001 111
011 111
010 111
110 111
111 111
101 111
100 111
000 110
001 110
011 110
010 110
110 110
111 110
101 110
100 110
000 010
001 010
011 010
010 010
110 010
111 010
101 010
100 010
000 011
001 011
011 011
010 011
110 011
111 011
101 011
100 011
000 001
001 001
011 001
010 001
110 001
111 001
101 001
100 001
000 000
001 000
011 000
010 000
110 000
111 000
101 000
100 000
+7
+5
+3
+1
−1
−3
−5
−7


Q
I



Poakis, “Digital Communications,” McGraw-Hill, 1983