Carrier Frequency Offset Estimation and Correction

Silicon DSP Corporation

Carrier Frequency and Symbol Clock Frequency Tolerance

17.3.9.4 Transmit center frequency tolerance

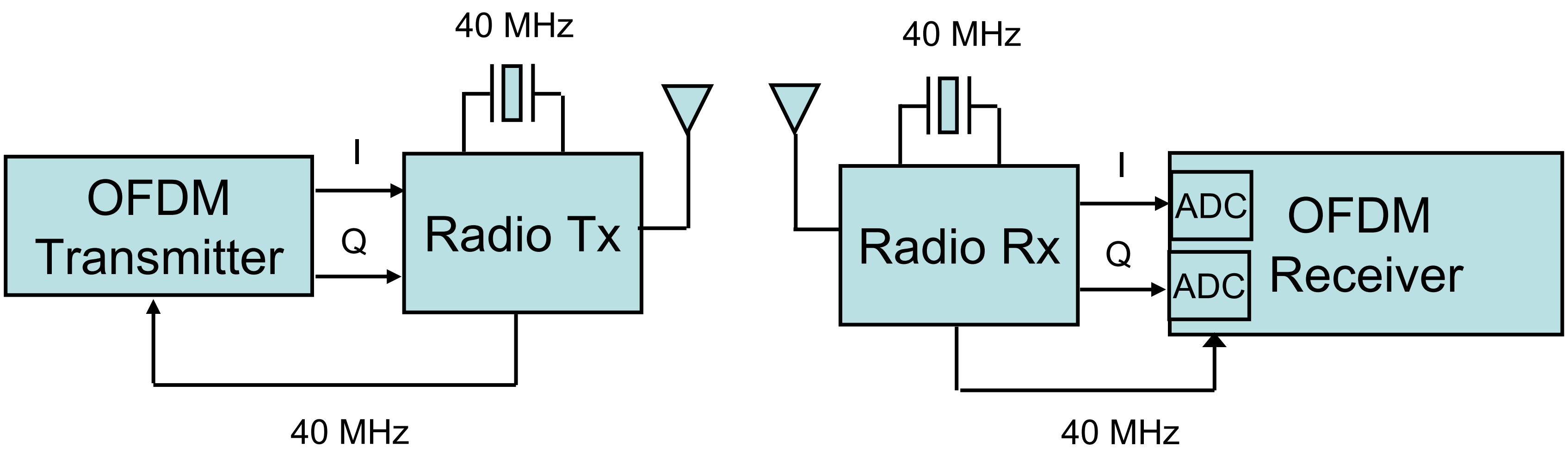
The transmitted center frequency tolerance shall be ± 20 ppm maximum. The transmit center frequency and the symbol clock frequency shall be derived from the same reference oscillator.

17.3.9.5 Symbol clock frequency tolerance

The symbol clock frequency tolerance shall be ± 20 ppm maximum. The transmit center frequency and the symbol clock frequency shall be derived from the same reference oscillator.

IEEE Std 802.11a-1999
(Supplement to IEEE Std 802.11-1999)

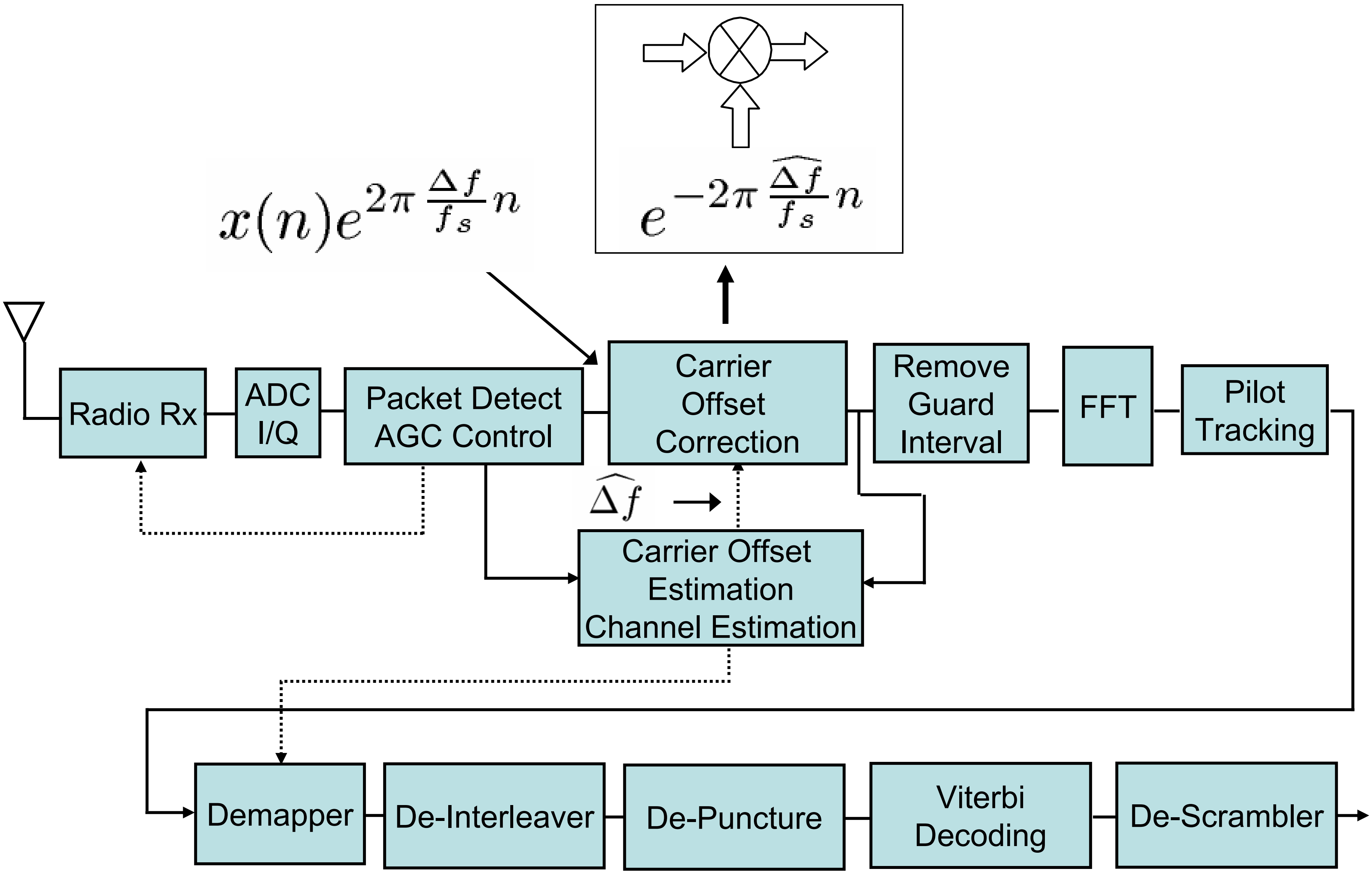
At 5 GHz can have a frequency offset between Tx and Rx of 200 kHz.


40 MHz
40 MHz
OFDM
Transmitter
I
Q
Radio Tx
Radio Rx
I
Q
ADC
ADC
OFDM
Receiver
40 MHz
40 MHz

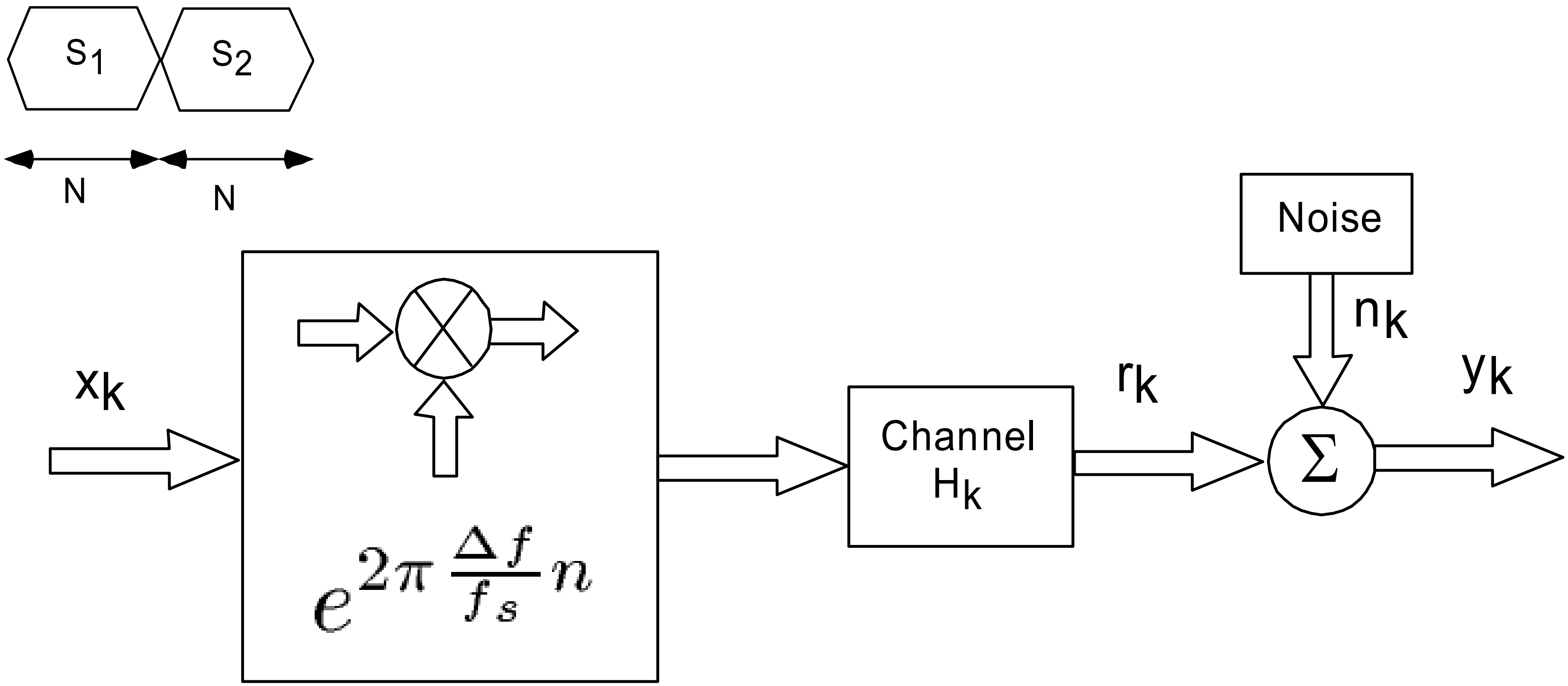
$$e^{j2\pi\Delta f t}$$

$$t_n = n\Delta t = \frac{n}{f_s}$$

$$e^{j2\pi\frac{\Delta f}{f_s}n}$$


$e^{-2\pi \frac{\widehat{\Delta f}}{f_s} n}$
$x(n)e^{2\pi \frac{\Delta f}{f_s} n}$
Radio Rx
ADC I/Q
Packet Detect AGC Control
Carrier Offset Correction
Remove Guard Interval
FFT
Pilot Tracking
$\widehat{\Delta f}$
Carrier Offset Estimation Channel Estimation
Demapper
De-Interleaver
De-Puncture
Viterbi Decoding
De-Scrambler

Algorithm for the Estimation of Carrier Offset Frequency



Paul Moose,"A Technique for Orthogonal Frequency Division Multiplexing Frequency Offset Correction," *IEEE Transcations on Communications*, Vol. 42, No. 10, October 1994

Transmitted Samples are DFT of Carriers:

$$x_n = \frac{1}{N}\sum_{k=-K}^{K} X_k e^{j2\pi nk/N}$$

$$N \geq 2K+1$$

k^{th} Carrier

Received Signal Through Channel

With Carrier Offset and No AWGN

Frequency Offset

$$r_n = \frac{1}{N}\sum_{k=-K}^{K} X_k H_k e^{j2\pi nk/N} e^{j2\pi \frac{\Delta f}{f_s} n}$$

Frequency Response at k^{th} Carrier

$$r_n = \frac{1}{N}\sum_{k=-K}^{K} X_k H_k e^{j2\pi n(k+\frac{\Delta f}{f_s}N)/N}$$

Define Carrier Offset Ratio

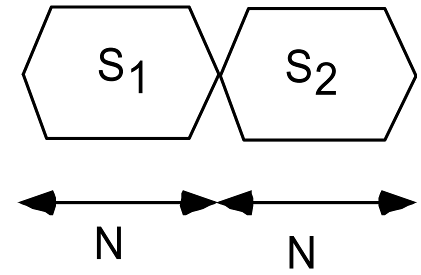
$$\epsilon = \frac{\Delta f}{f_s} N$$

$$r_n = \frac{1}{N}\sum_{k=-K}^{K} X_k H_k e^{j2\pi n(k+\epsilon)/N}$$

Carrier Spacing: $f_c = \frac{f_s}{N}$

Ratio of Offset to Carrier Spacing

$$\epsilon = \frac{\Delta f}{f_c}$$



$$R_{1k} = \sum_{n=0}^{N-1} r_n e^{-j2\pi nk/N}$$

$$R_{2k} = \sum_{n=N}^{2N-1} r_n e^{-j2\pi nk/N}$$

$$r_n = \frac{1}{N} \sum_{k=-K}^{K} X_k H_k e^{j2\pi n(k+\epsilon)/N}$$

$$r_{n+N} = r_n e^{j2\pi\epsilon}$$

$$y_k = r_k + n_k$$

$$Y_{1k} = R_{1k} + W_{1k}$$

$$Y_{2k} = R_{1k} e^{j2\pi\epsilon} + W_{2k}$$

Maximum Likelihood Estimation

$$\widehat{\epsilon} = \frac{1}{2\pi} \tan^{-1} \left[\frac{\sum_{k=-K}^{K} Im(Y_{2k} Y_{1k}^{*})}{\sum_{k=-K}^{K} Re(Y_{2k} Y_{1k}^{*})} \right]$$

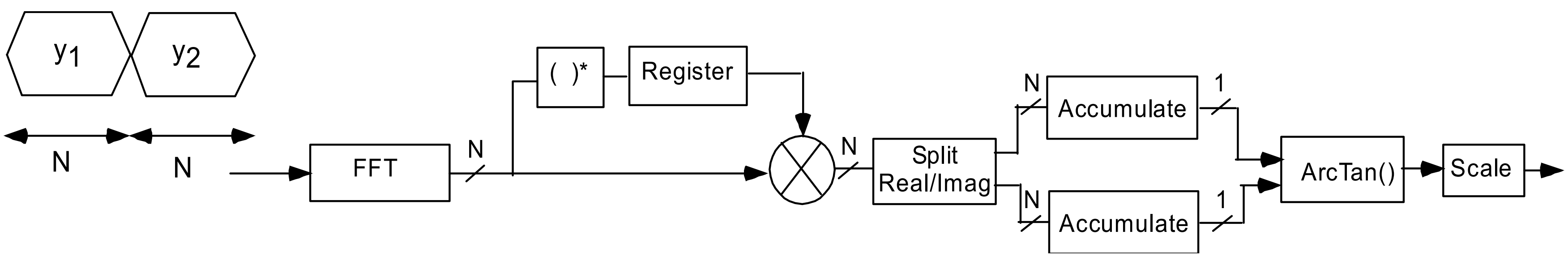
No Noise:

$$Y_{2k} Y_{1k}^{*} = |R_{1k}|^{2} e^{j2\pi\epsilon}$$

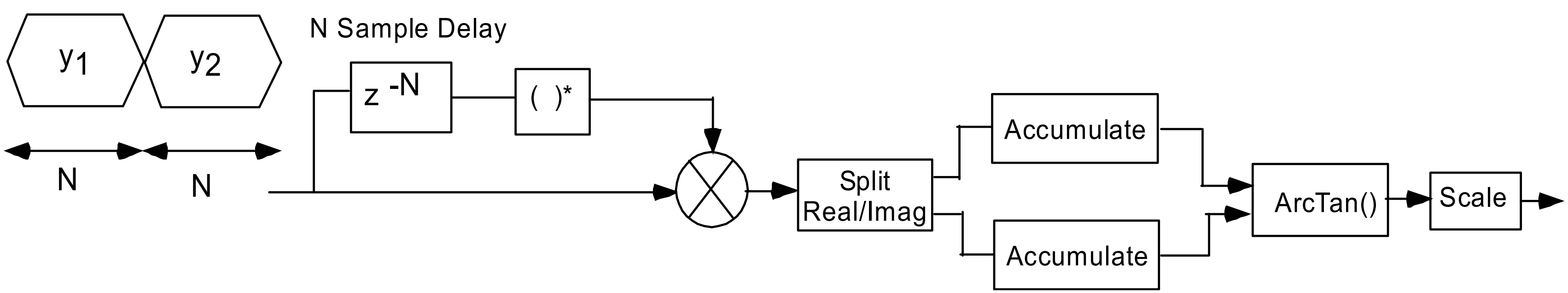
$$\widehat{\epsilon} = \frac{1}{2\pi} \tan^{-1} \left[\frac{2K sin(2\pi\epsilon)}{2K cos(2\pi\epsilon)} \right] \longrightarrow \widehat{\epsilon} = \epsilon$$

Implementation

Frequency Domain



Time Domain



For example of above time domain autocorrelation method see Eberle, et. al, Scalable Digital OFDM Transceiver …, *IEEE Journal of Solid –State Circuits*, Vol. 36, No. 11, November 2001.

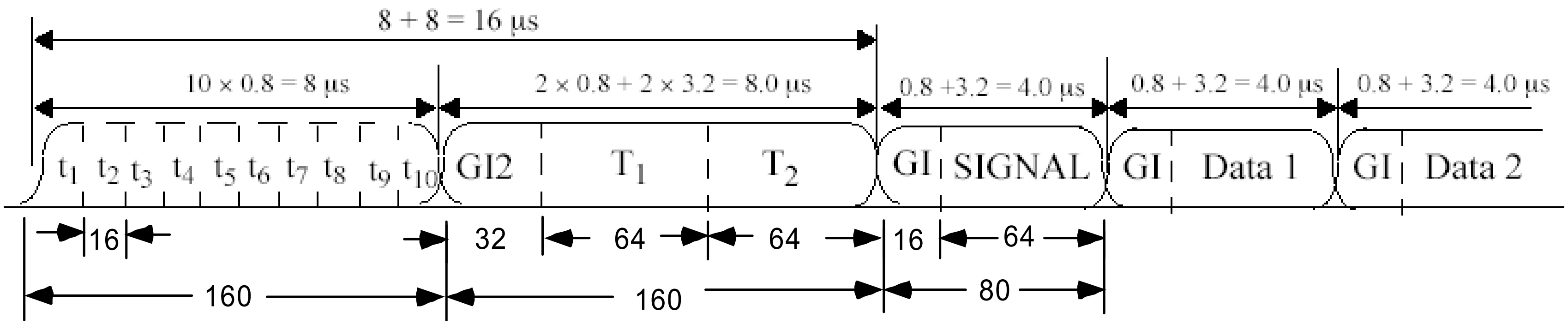
They also use CORDIC for Arc Tan.

Additive Noise and Carrier Offset Estimation

$$\sigma_\epsilon^2 = E(\widehat{\epsilon} - \epsilon)^2 = \frac{1}{4\pi^2 N} \frac{N_0}{E_c}$$

E_c	Each Sub Carrier Energy
N_0	One-sided spectral density of AWGN noise
N	Number of Carriers

With noise and long duration packet need to correct for residual carrier offset frequency using pilot tracking.



• Moose's algorithm can estimate carrier offset up to half the carrier spacing.

• For long symbols, we have 64 samples at 20 MHz, so carrier spacing is 20MHz/64=312.5kHz.

• So using long symbol can estimate carrier offset up to 150 kHz.

• For 16 sample short symbols that are duplicates, at 20 MHz sampling rate the carrier spacing is: 20MHz/16=1.25 MHz.

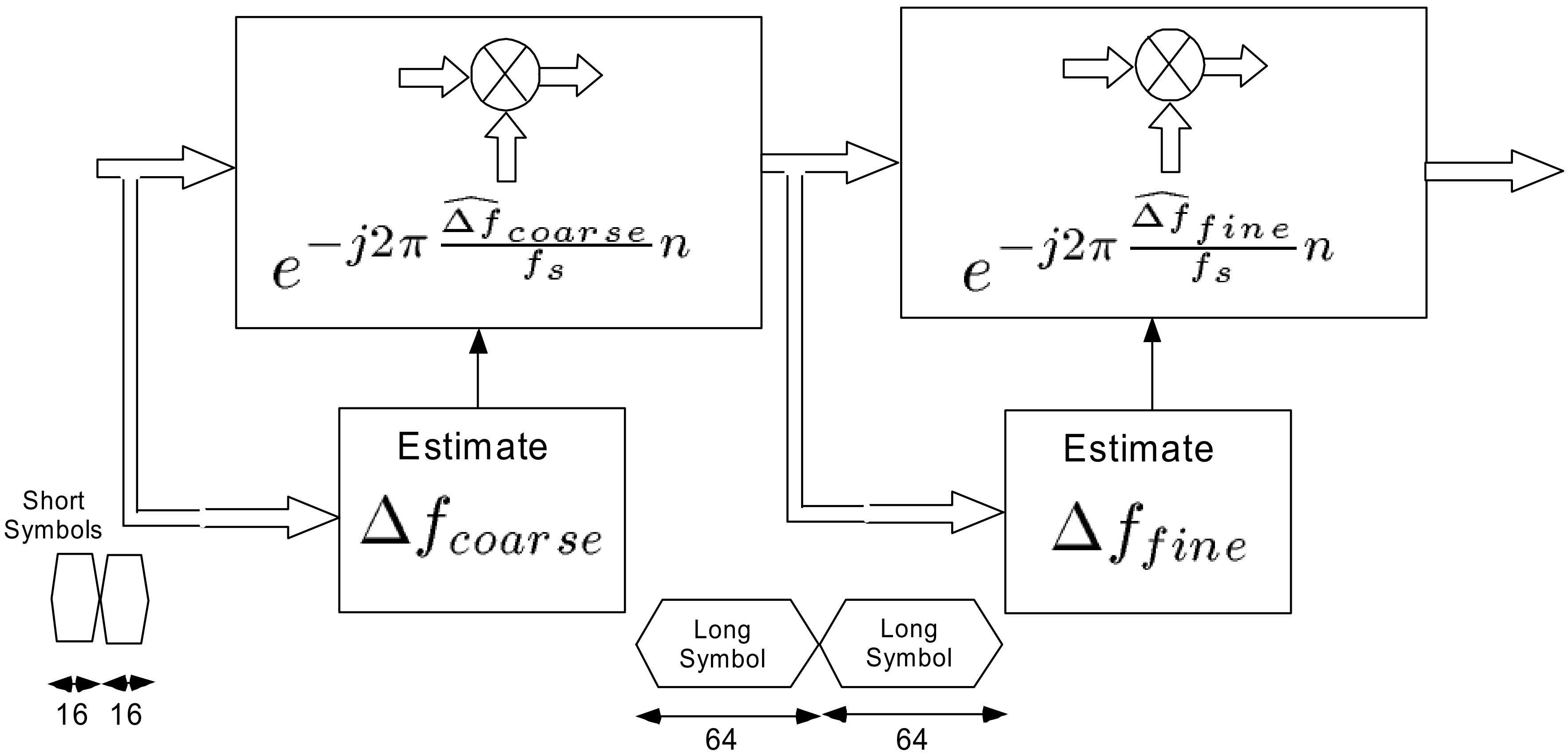
• So using the short symbols we can estimate up to 625 KHz.

• So for 802.11a use short symbols for coarse and long symbol for fine frequency carrier offset estimation.

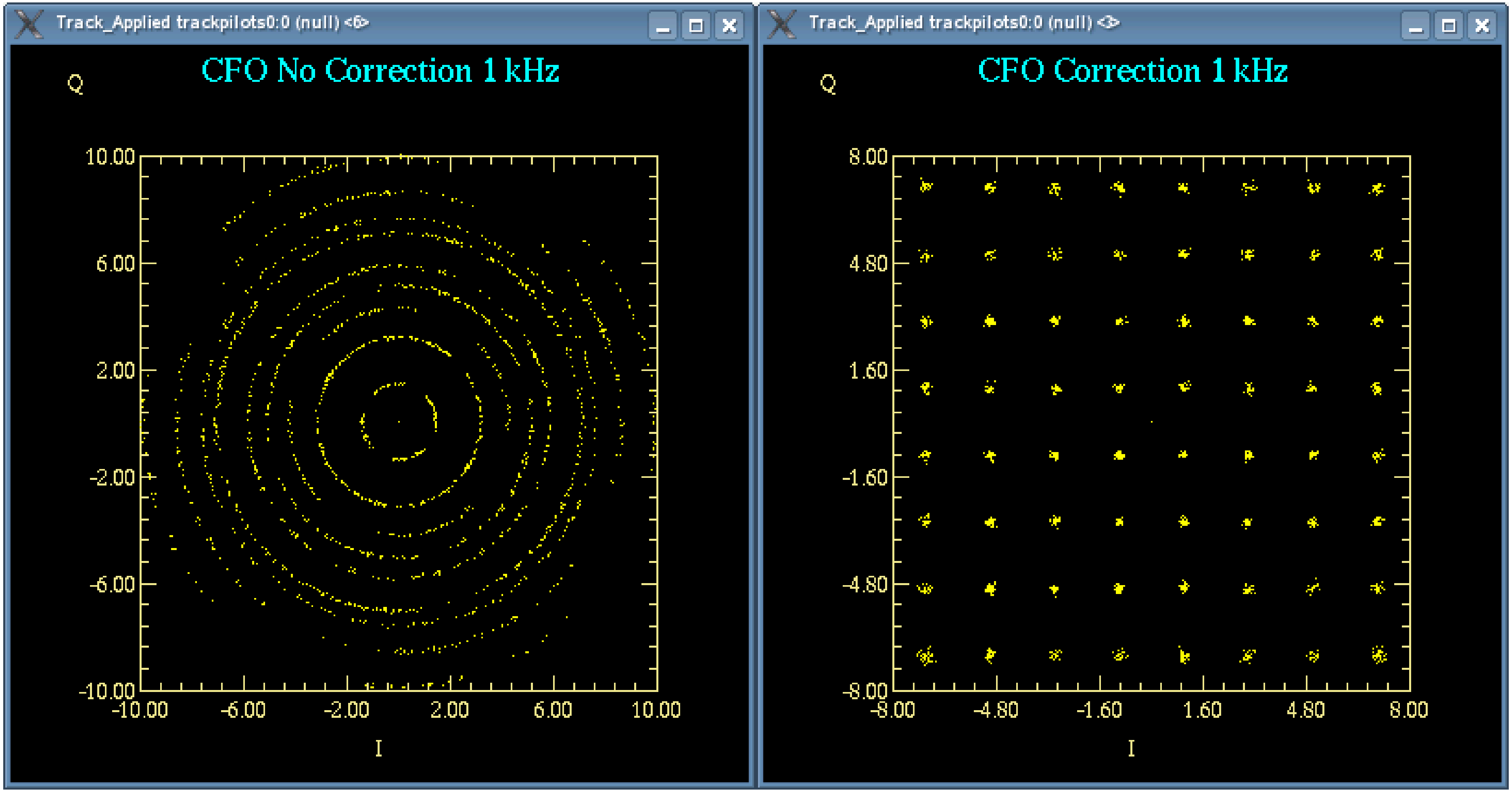
Frequency Carrier Offset

$$\epsilon = \frac{\Delta f}{f_c}$$

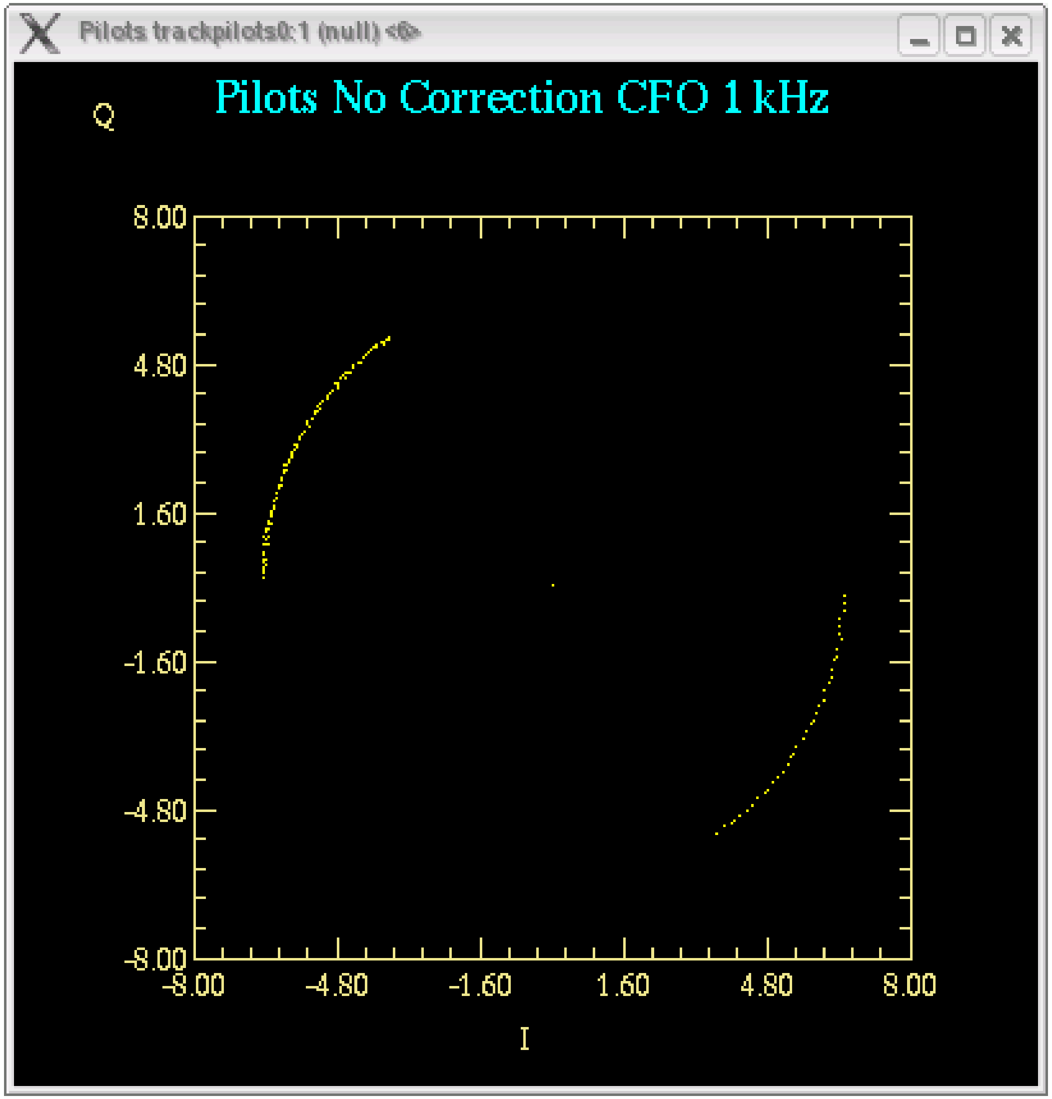
Carrier Spacing



$e^{-j2\pi \frac{\widehat{\Delta f}_{coarse}}{f_s} n}$
$e^{-j2\pi \frac{\widehat{\Delta f}_{fine}}{f_s} n}$
Estimate
Δf_{coarse}
Estimate
Δf_{fine}
Short Symbols
16 16
Long Symbol
Long Symbol
64
64

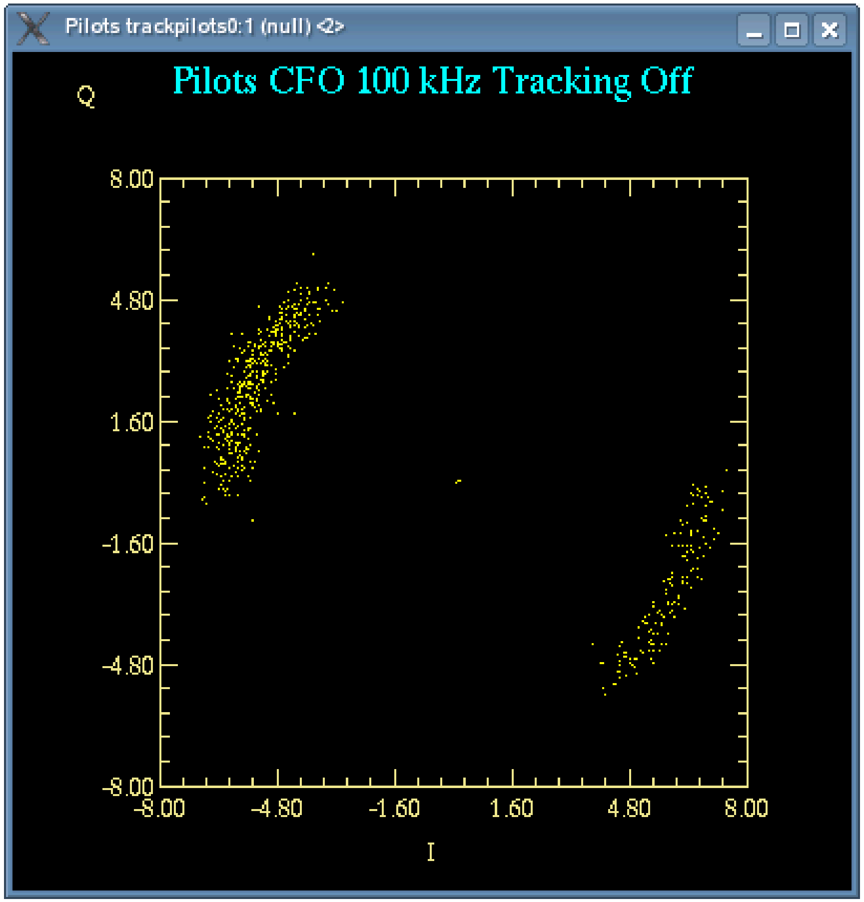


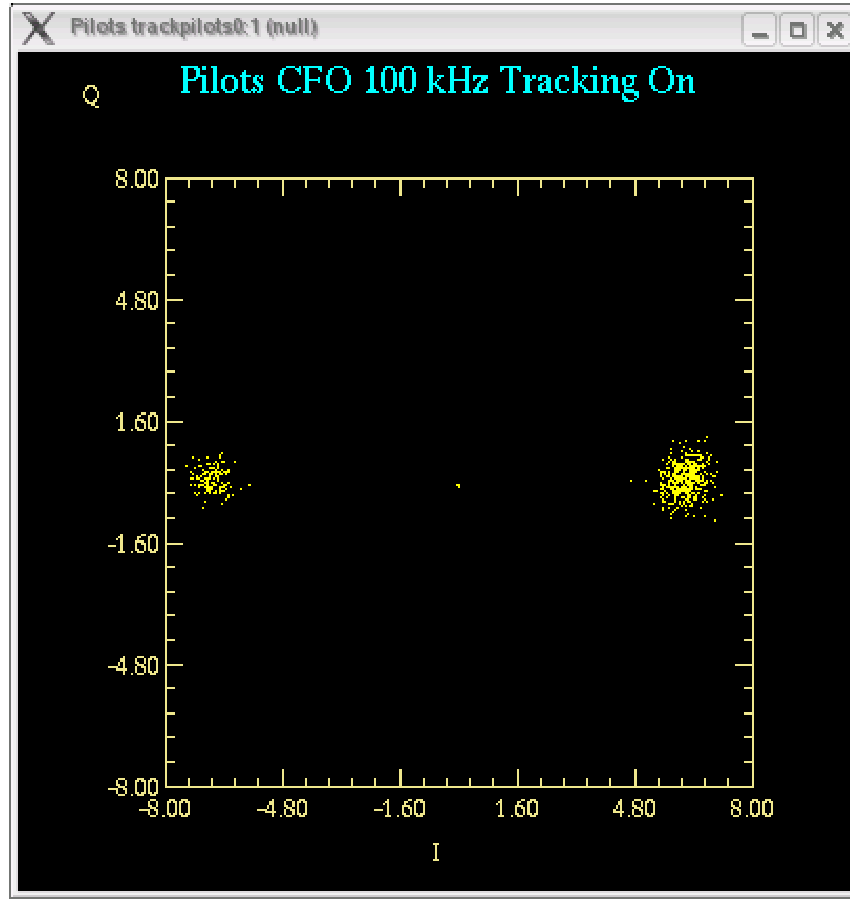
1000 Bytes SNR= 40dB



1000 Bytes SNR =40 dB

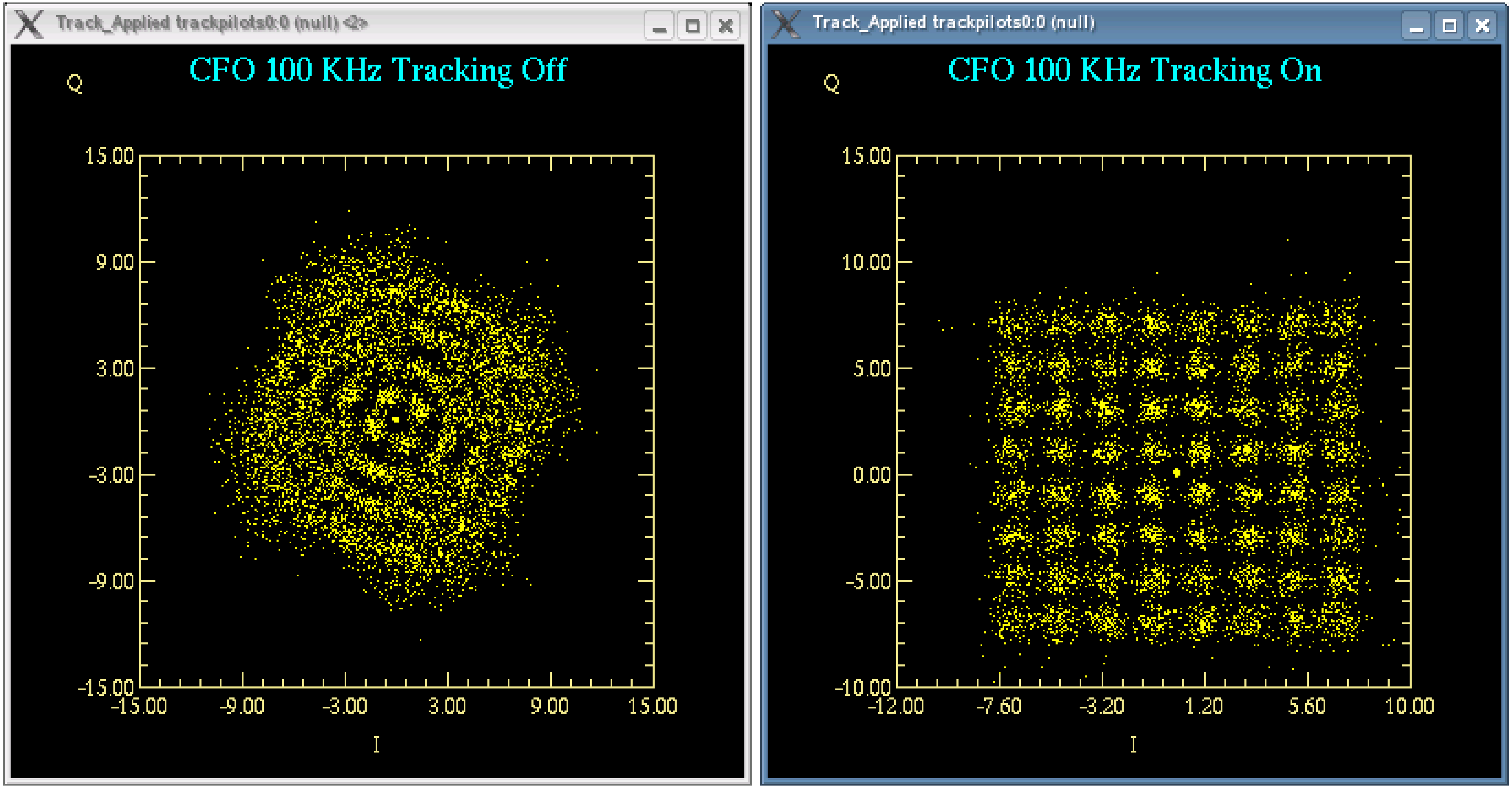
Residual CFO Correction with Pilot Tracking





4000 Bytes SNR=24 dB

Residual CFO Correction with Pilot Tracking



4000 Bytes SNR=24 dB